Lattice Coding Increases Multicast Rates for Gaussian Multiple-Access Networks

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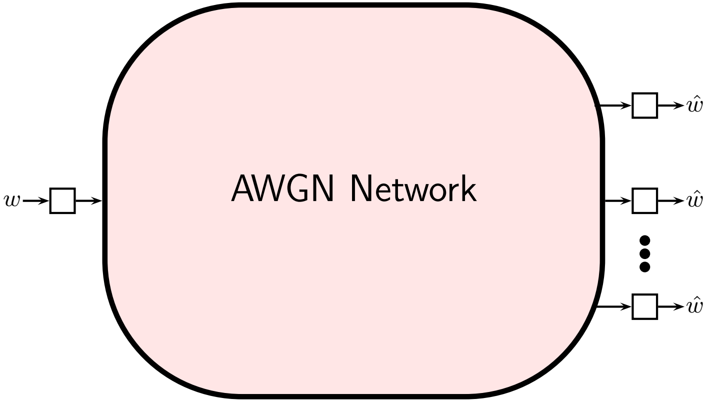
Multicasting over AWGN Networks



- Single sender

Multicasting problem only well understood for point-to-point channel networks.

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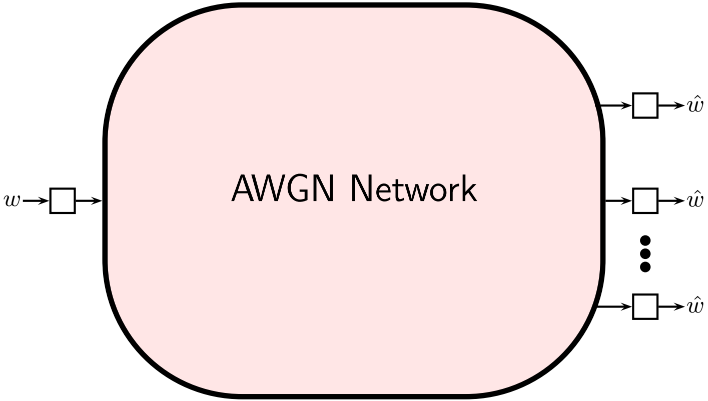

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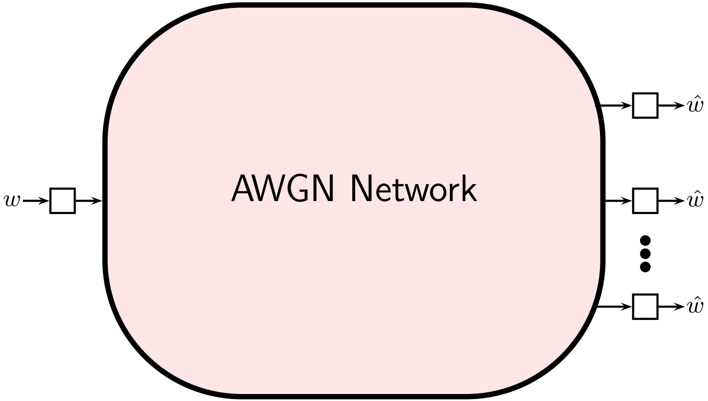

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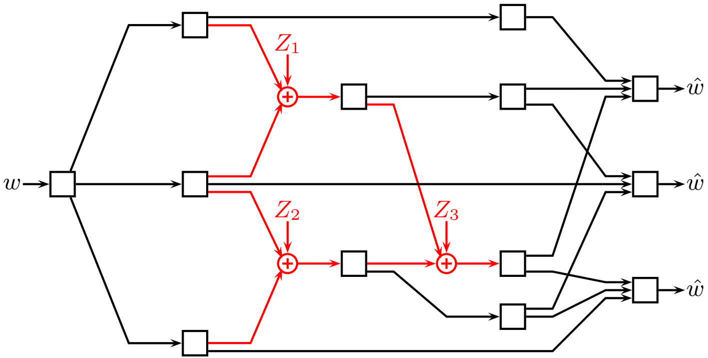
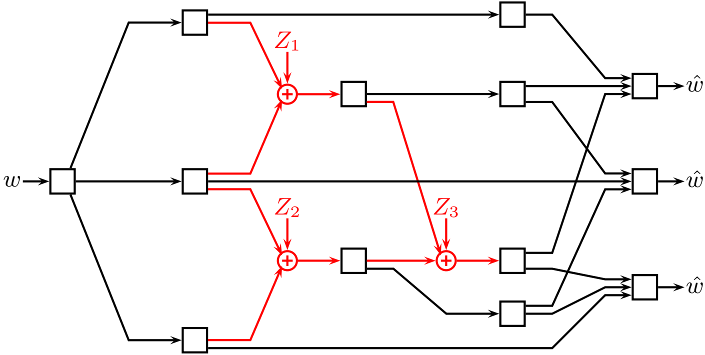
Multicasting over Gaussian Multiple-Access Networks



- Gaussian MACs

Multicasting over Gaussian Multiple-Access Networks



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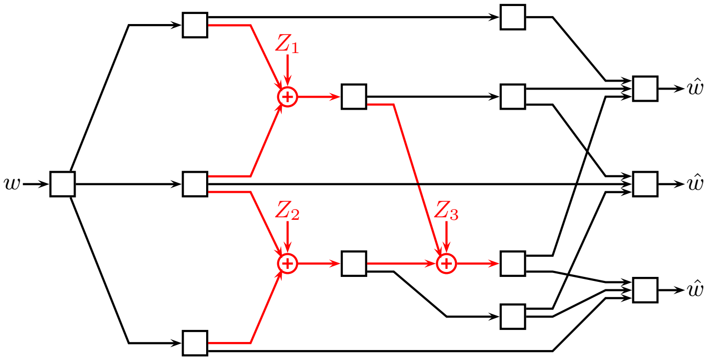

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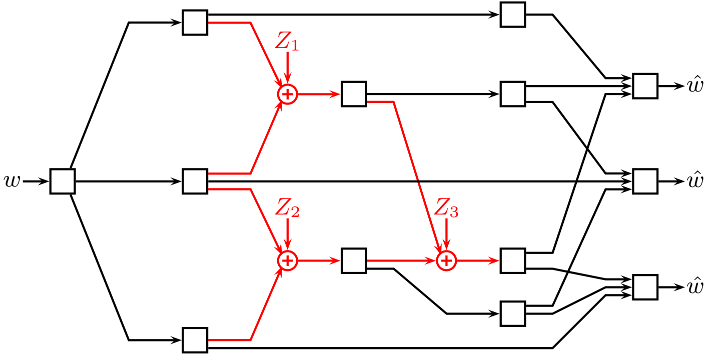

- Gaussian MACs
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- Want a nice reduction to a point-to-point network.
- Usual solution: replace MACs with bit pipes using a MAC code.

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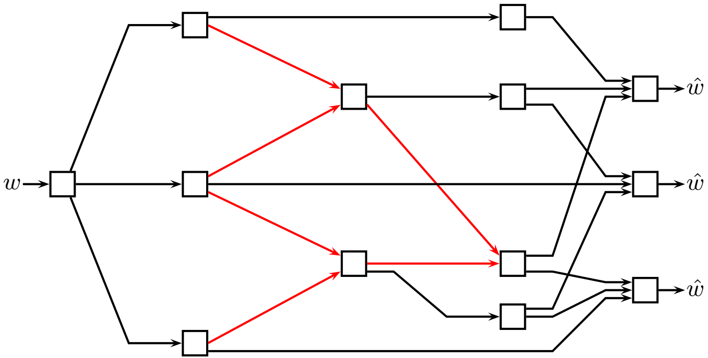

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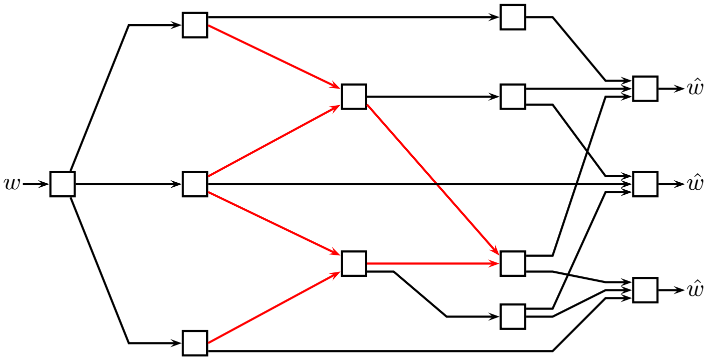

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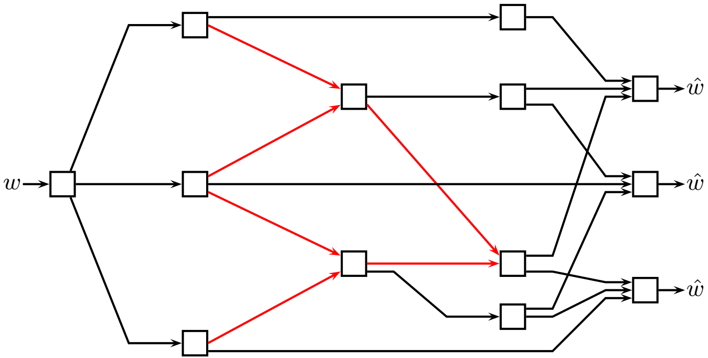

- Bit pipe network
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- Reduction ignores linear functions performed by MACs.
- MAC can do network coding with structured random codes.

Overview

Main ideas for this talk:

$\Longrightarrow$ (*Structured*) random coding technique that exploits structural gain not beamforming gain.

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$\Longrightarrow$ (*Structured*) random coding technique that exploits structural gain not beamforming gain.

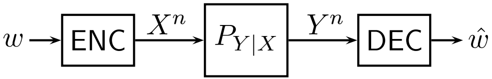
$\Longrightarrow$ New relaying strategy that allows relays in a network to reliably "compute-and-forward" functions of messages.

$\Longrightarrow$ New (achievable) multicast rates for AWGN networks with multiple-access components.

Outline

1. Background: Random Coding Theorems
2. Motivating Example
3. Multicasting over AWGN Networks
   a. Problem Statement
   b. Coding Theorem
   c. Proof Ideas
4. Extensions

Point-to-Point Communication



- Capacity given by:

$$C = \max_{p(x)} I(X;Y)$$

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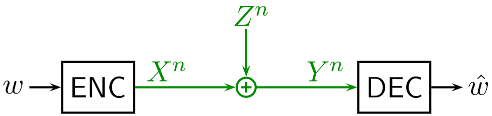

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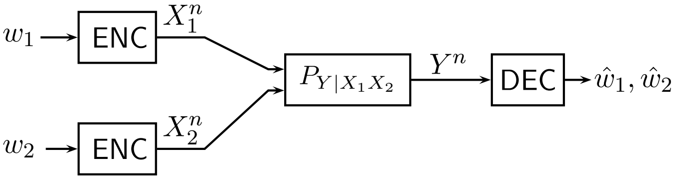

- Capacity region is the convex closure of all rate pairs satisfying:

$$R_1 < I(X_1;Y|X_2)$$
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  for some product distribution $p(x_1)p(x_2)$.
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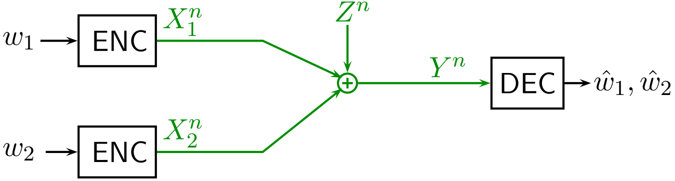

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Natural Extension to General Networks

- We expect capacity results in terms of mutual informations of some distributions. For example:

$$\max_{p(x_1,x_3)p(x_2|x_3)p(\textbf{stuff})} \min\{I(X_1, \textbf{stuff}; Y_1|X_3) + I(\textbf{stuff}; Y_3), \\ I(X_1, X_2; Y_4), I(X_3; Y_1, \textbf{things})\}$$

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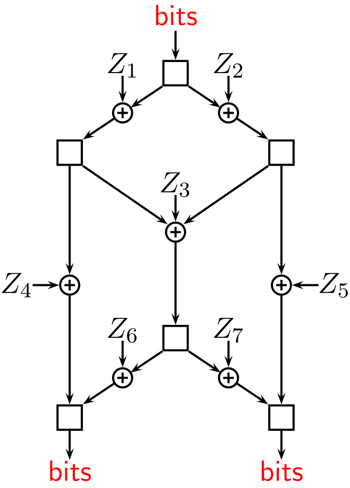
- Usual Achievability Proof: Draw codewords i.i.d. from desired distributions (as well as some very nice generalizations of this).
- Is focusing on the distributions enough? Do we just need better converses?
- No! These techniques fail (in expectation) as they do not exploit structure.

Motivating Example: "AWGN Butterfly"

- Drawing codewords i.i.d. from specified distributions is insufficient to prove network capacity theorems in expectation.



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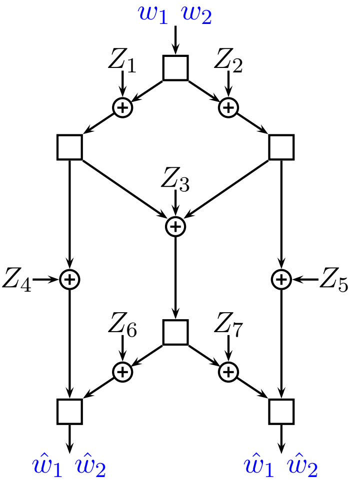

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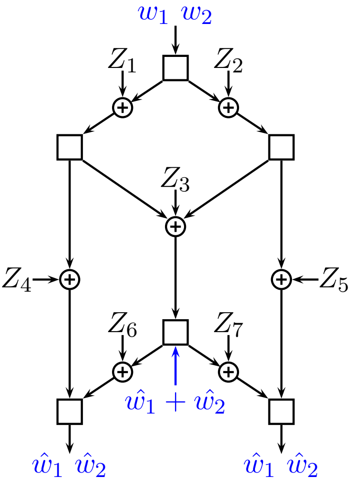

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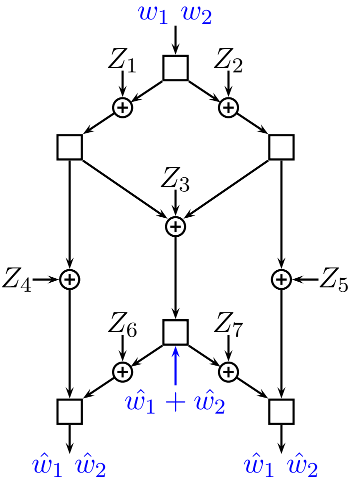
- Really just need sum on center path.
- Want to benefit from MAC's addition for structural gain.



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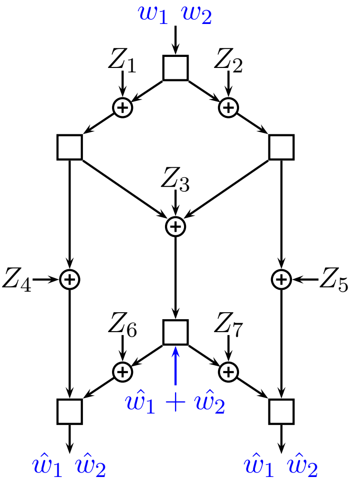

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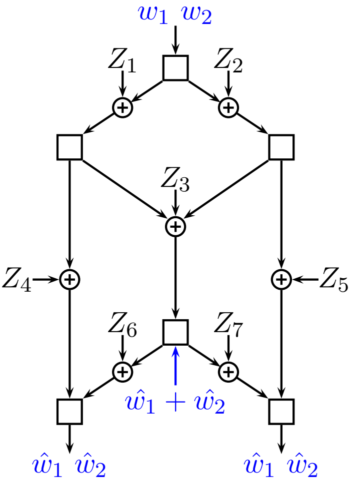

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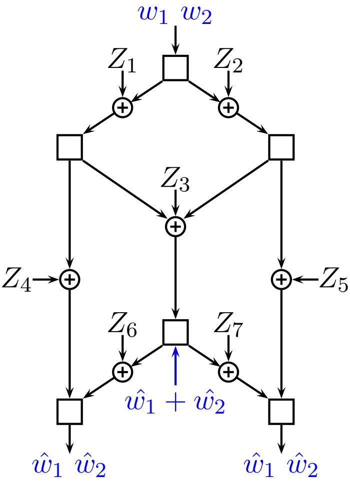

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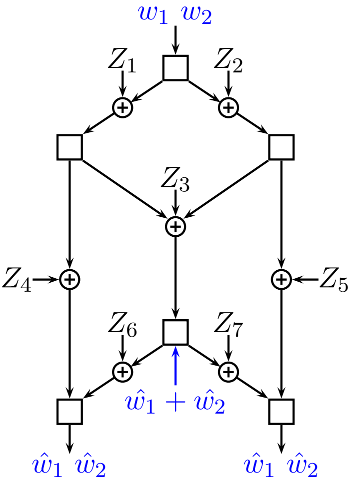
Really want to "compute-and-forward"!

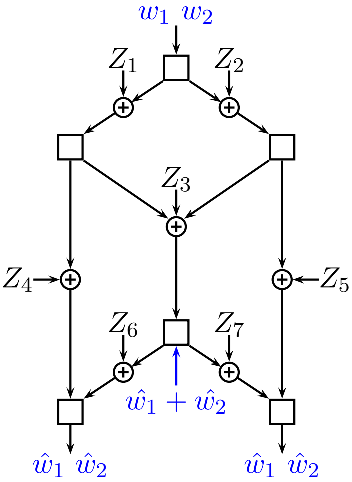


Performance Comparison

For equal per user SNR:

- Decode-and-forward multicasting rate:

$$R = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right) + \frac{1}{4}\log_2\left(1 + \frac{2P}{N}\right)$$



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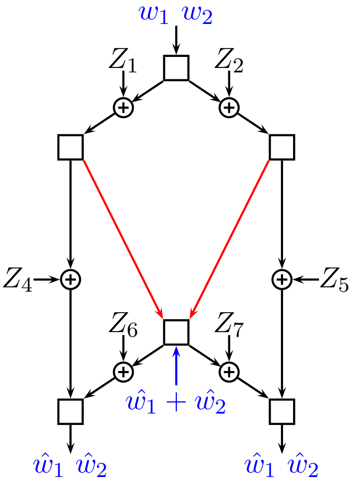

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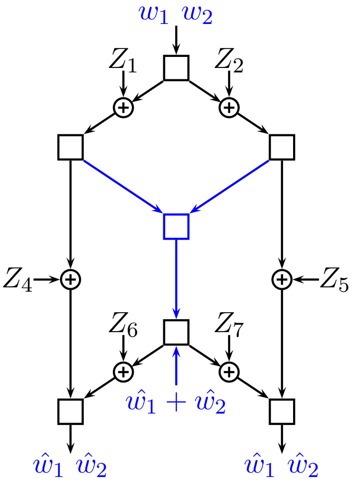
- Compute-and-forward achieves:

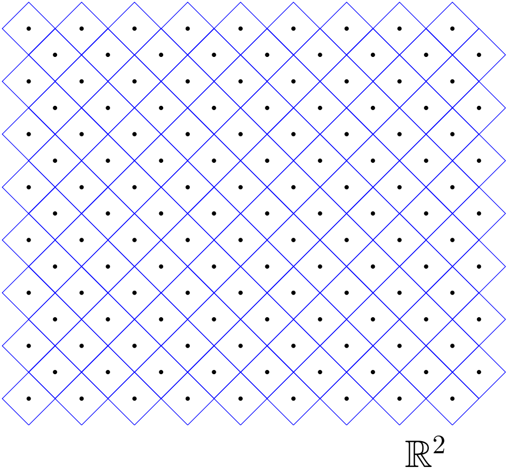
$$R = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right) + \frac{1}{2}\log_2\left(\frac{1}{2} + \frac{P}{N}\right)$$

- Compute-and-forward relies on lattices.



Lattice Basics

- Lattice is a linear tiling of $\mathbb{R}^n$



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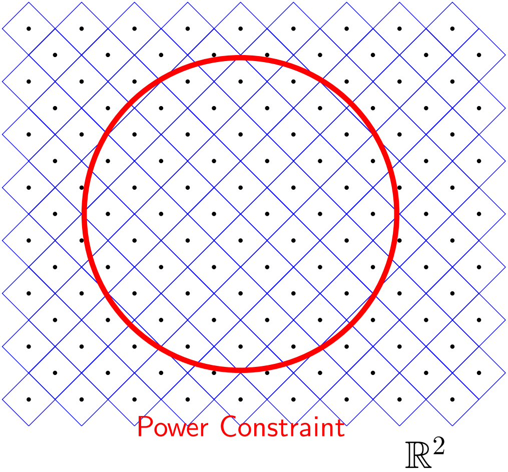

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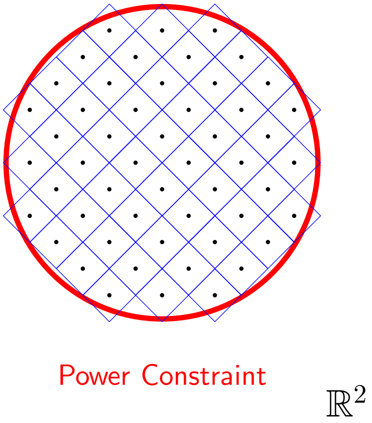

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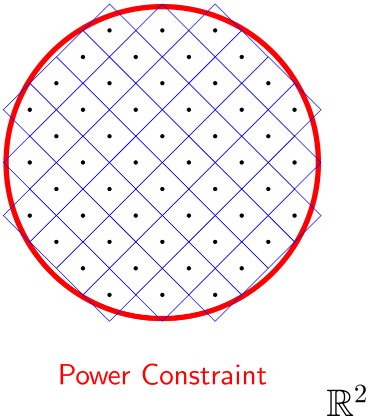
- Urbanke-Rimoldi '98: $\exists$ lattices that achieve AWGN capacity with ML decoding



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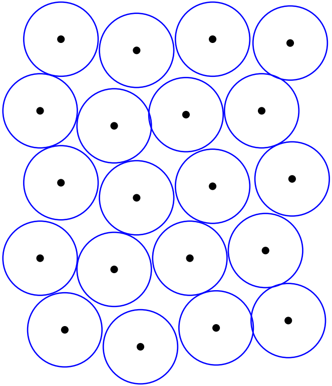

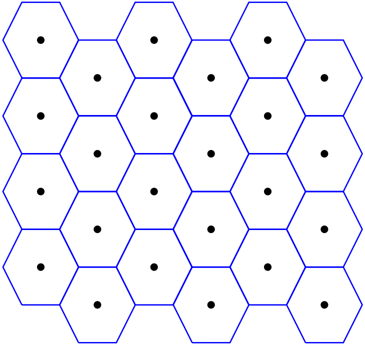

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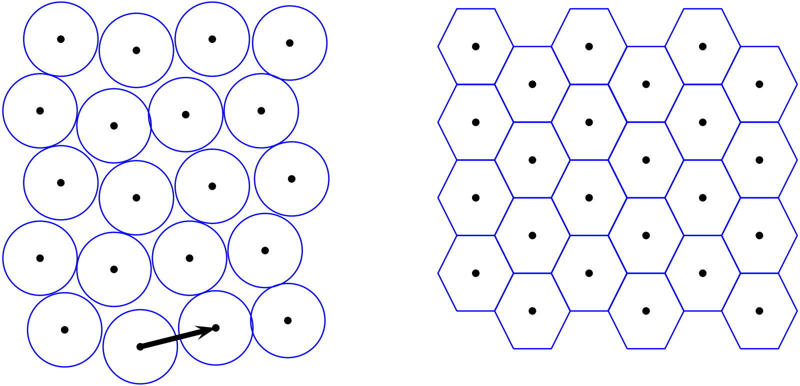

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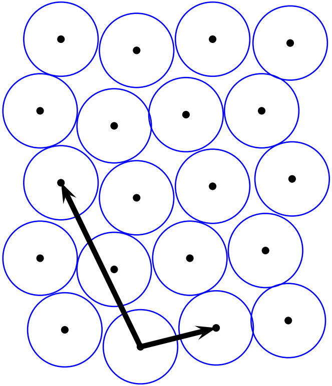

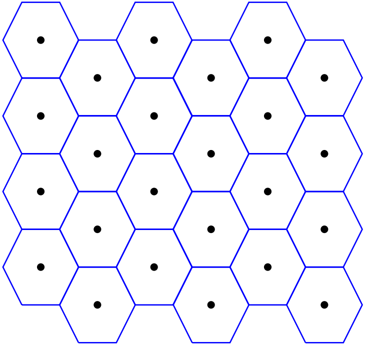

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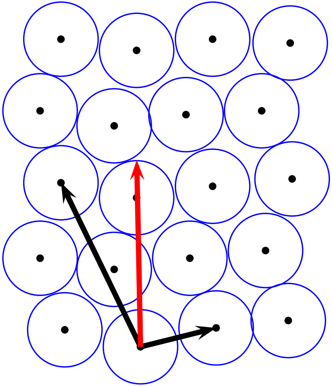

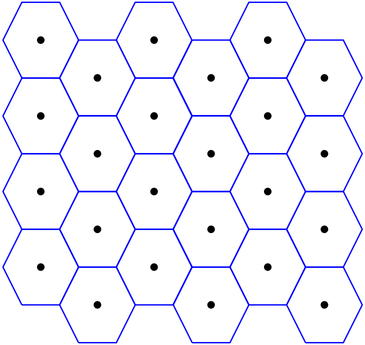

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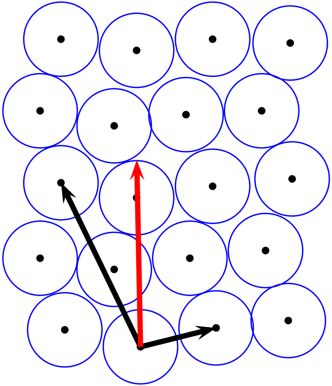

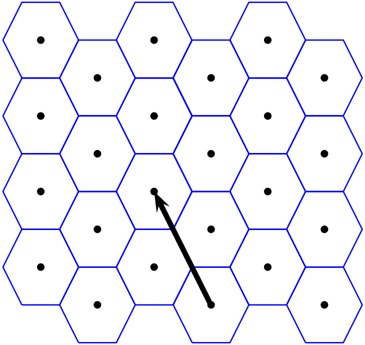

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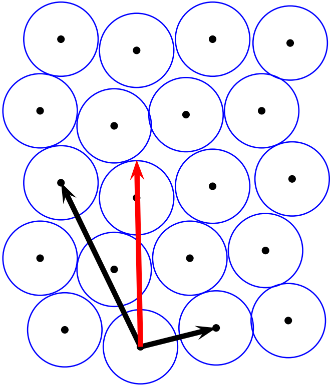

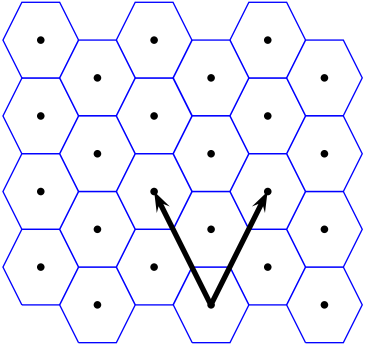

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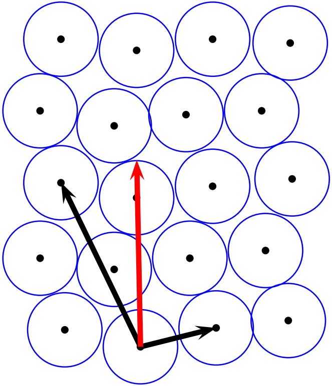

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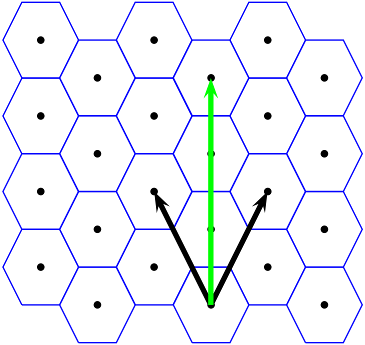

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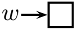
Problem Statement



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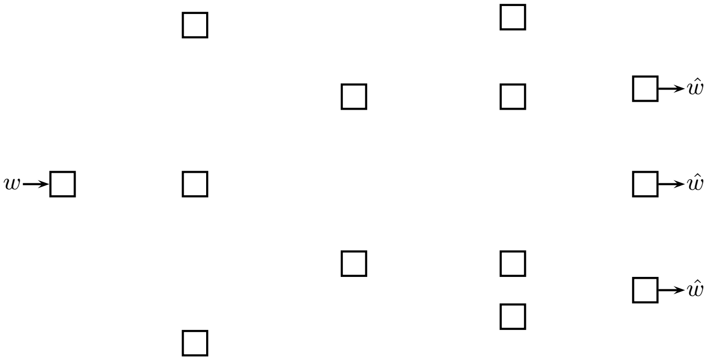

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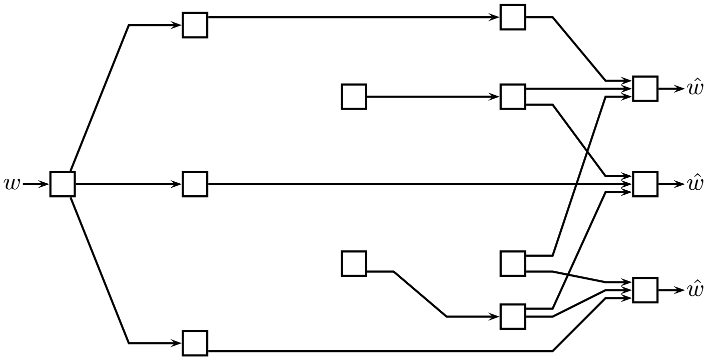

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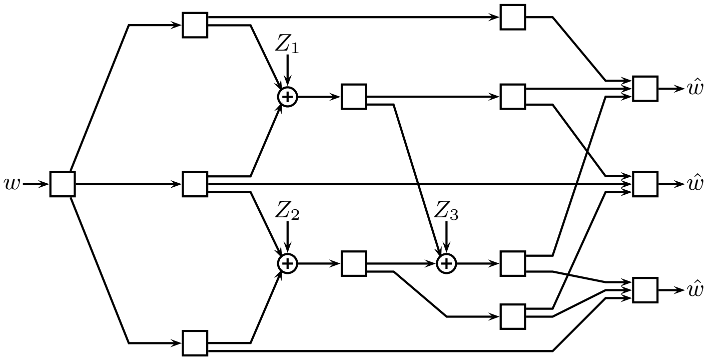

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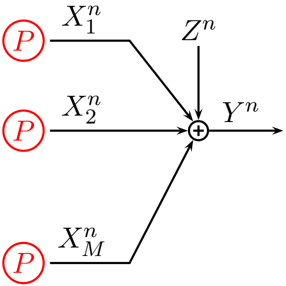
Channel Model Details

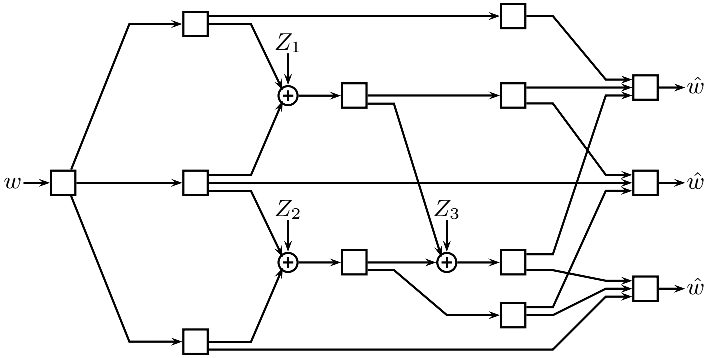


- i.i.d. additive Gaussian noise: $Z \sim \mathcal{N}(0, N_j)$
- Same transmit power constraint:

$$\frac{1}{n}\sum_{i=1}^{n}(x_j[i])^2 \leq P$$

- Channel quality controlled by noise variance.
- Scheme for different user transmit powers at the end of the talk...

Coding Theorem: New Achievable Rates

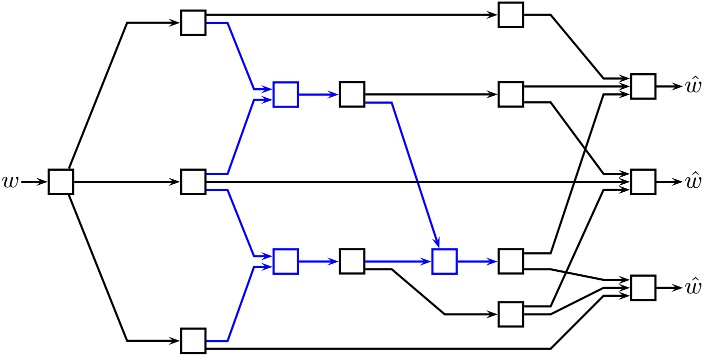


- Reduction to bit pipe network

Theorem

Any multicast rate achievable on the resulting network is achievable on the original network using a compute-and-forward scheme.

Coding Theorem: New Achievable Rates



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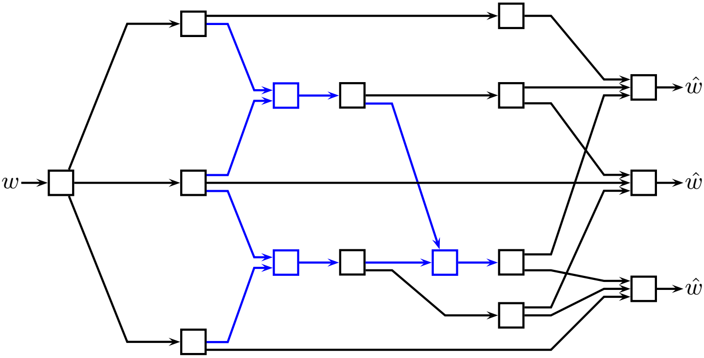

- Reduction to bit pipe network
- MACs become nodes
- Links have rate $\frac{1}{2}\log_2\left(\frac{1}{M_j}+\frac{P}{N_j}\right)$

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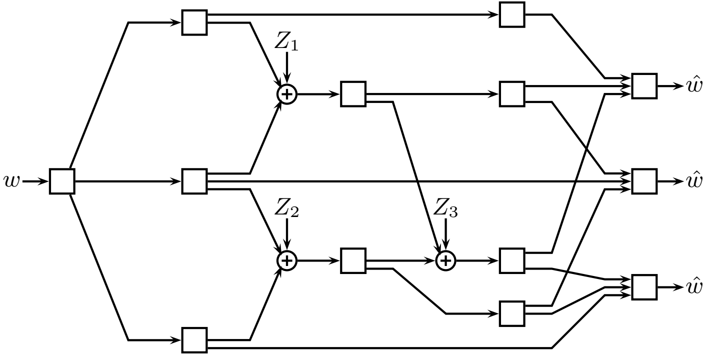
Any multicast rate achievable on the resulting network is achievable on the original network using a compute-and-forward scheme.

Random Coding: Interference Between Flows



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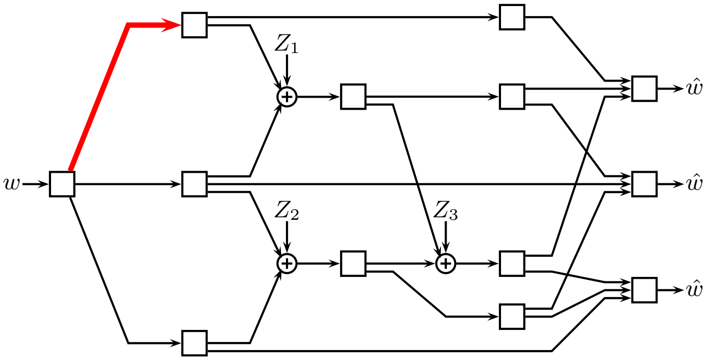

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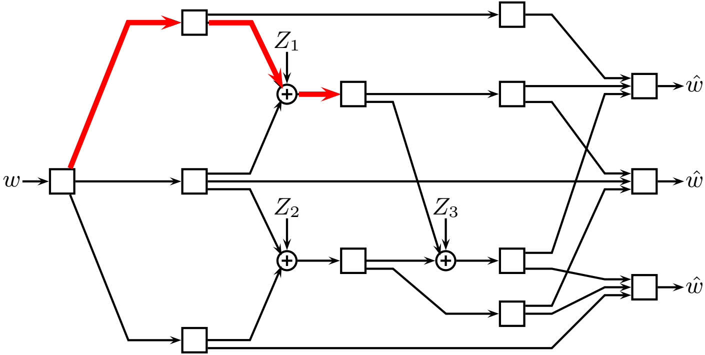

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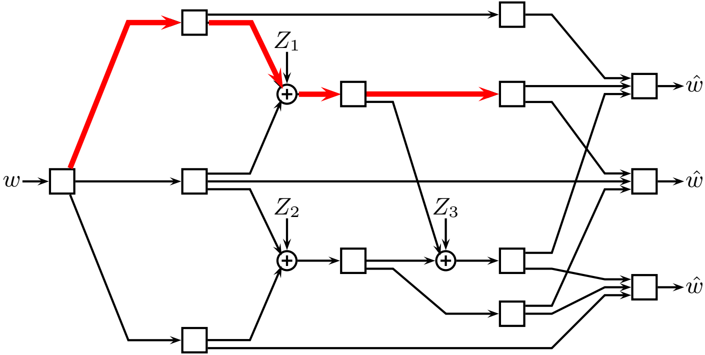

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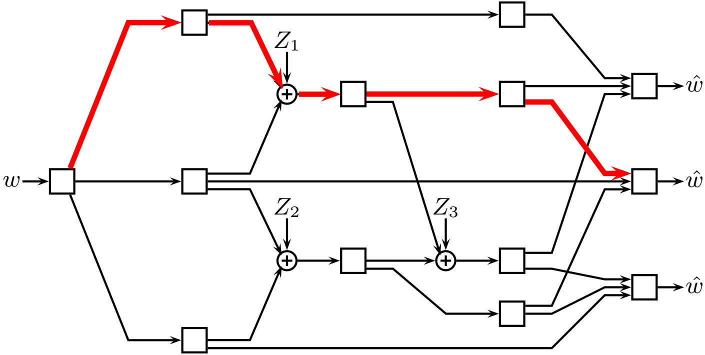

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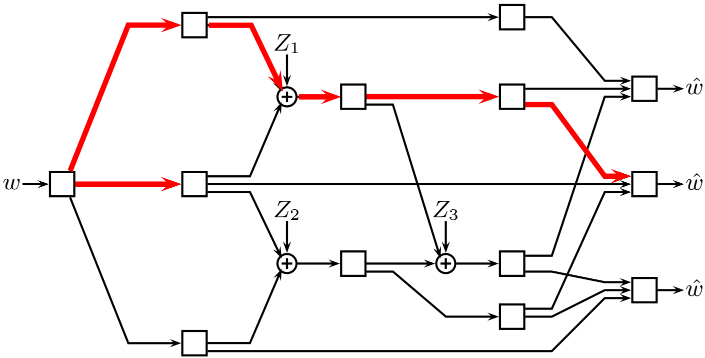

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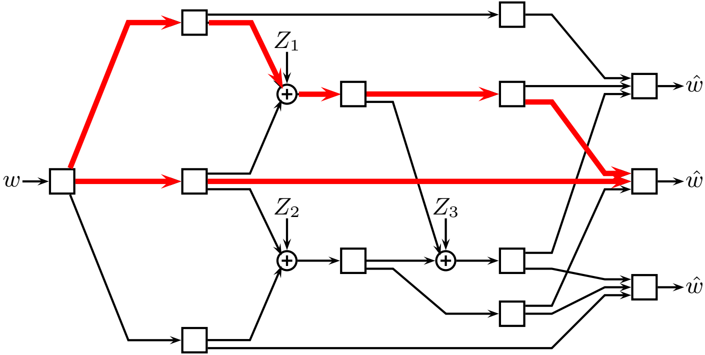

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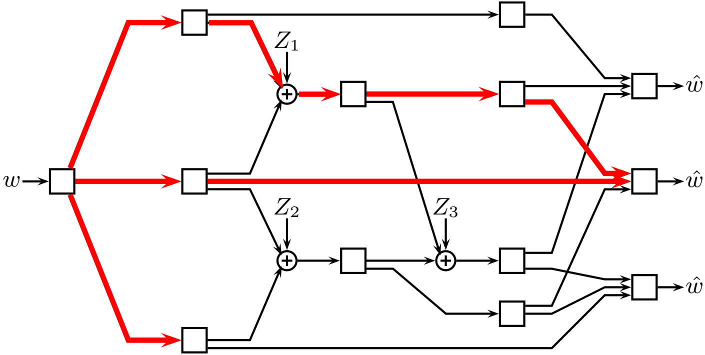

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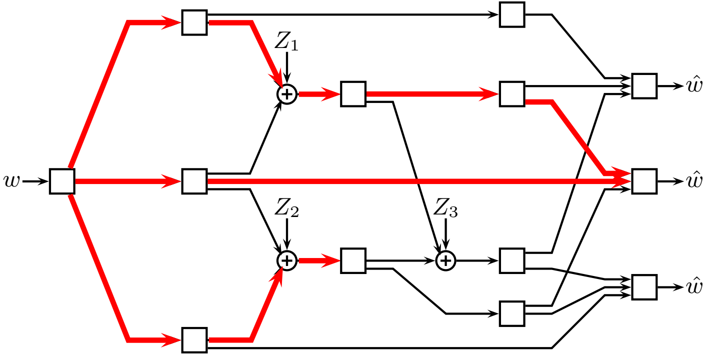

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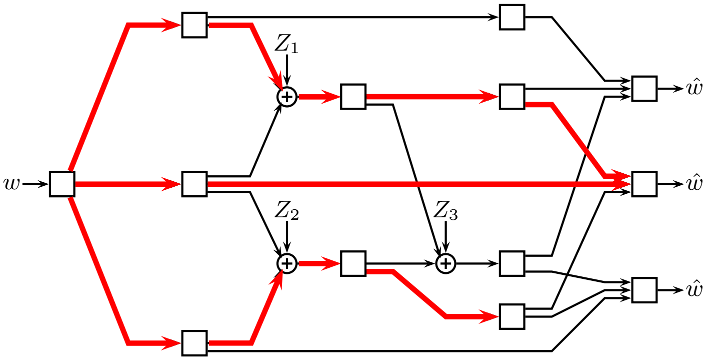

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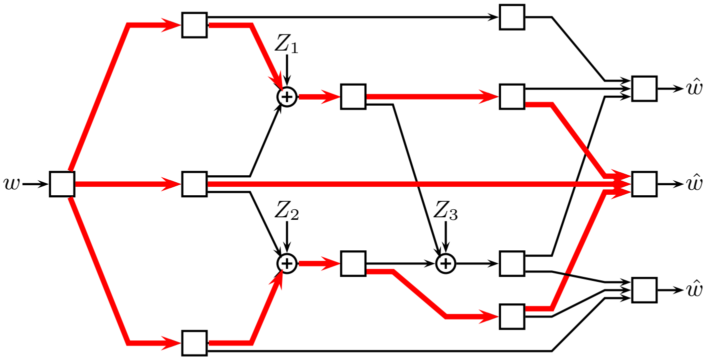

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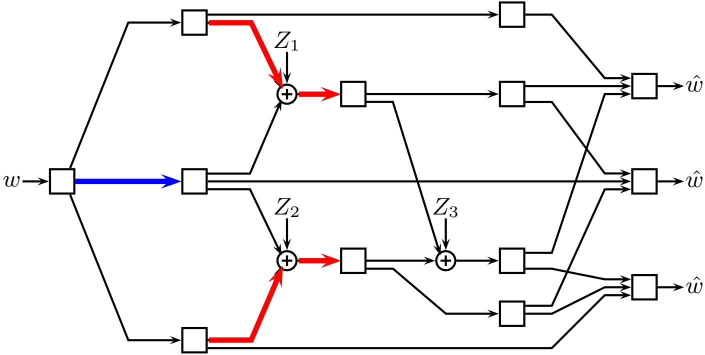

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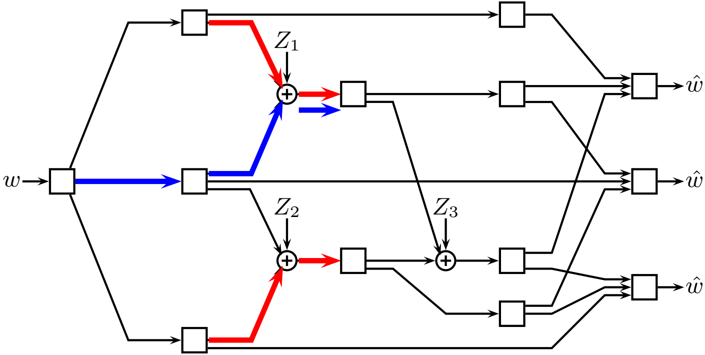

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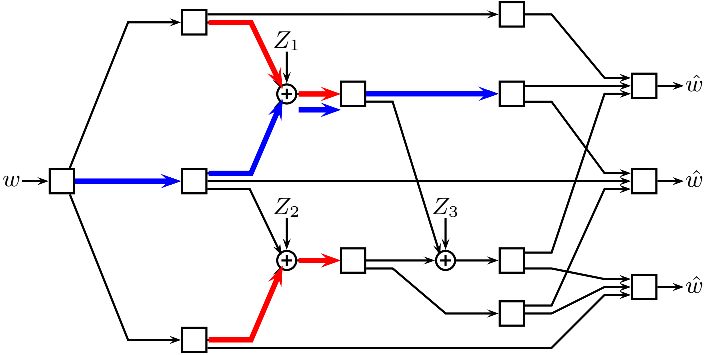

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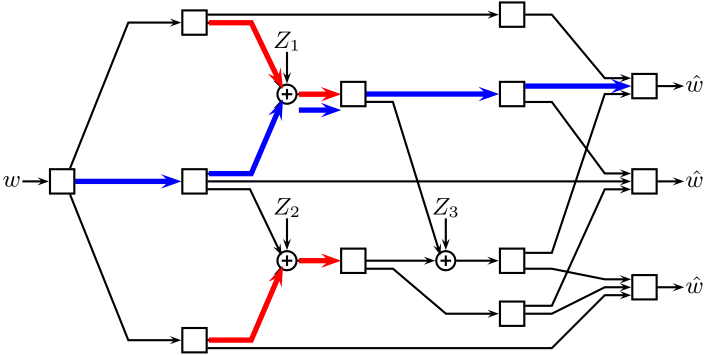

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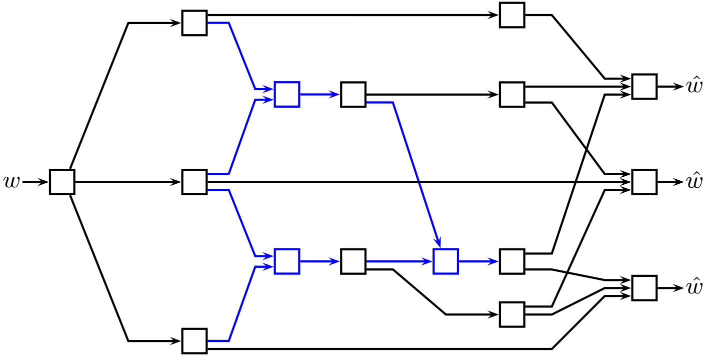
Compute-and-Forward: No Interference Between Flows



- Calculate flows as if MACs are interference free!
- MAC constraints are only: $R_j < \frac{1}{2}\log_2\left(\frac{1}{M_j} + \frac{P}{N_j}\right)$

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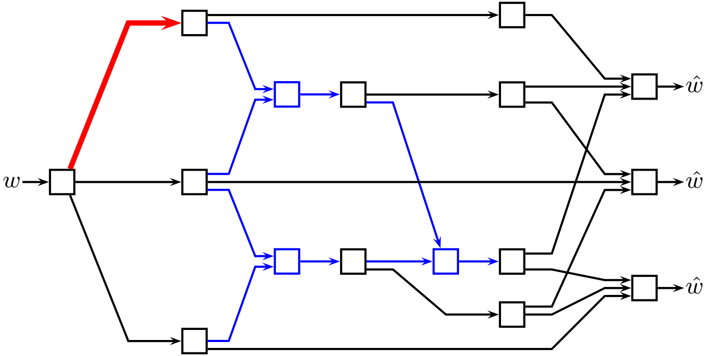

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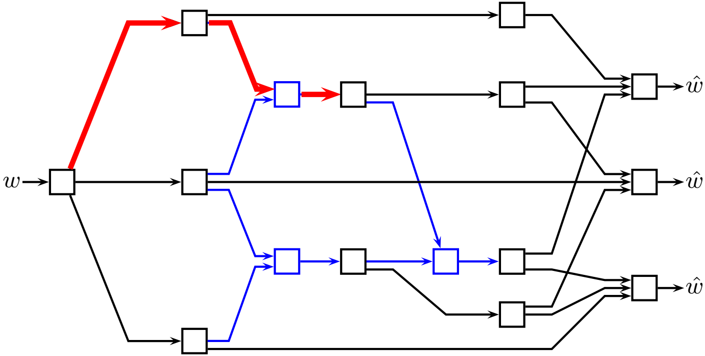

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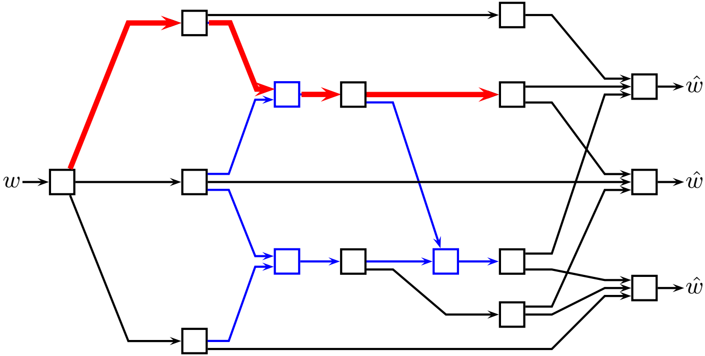

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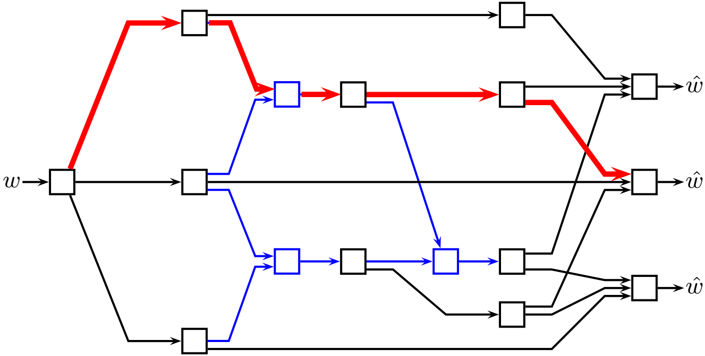

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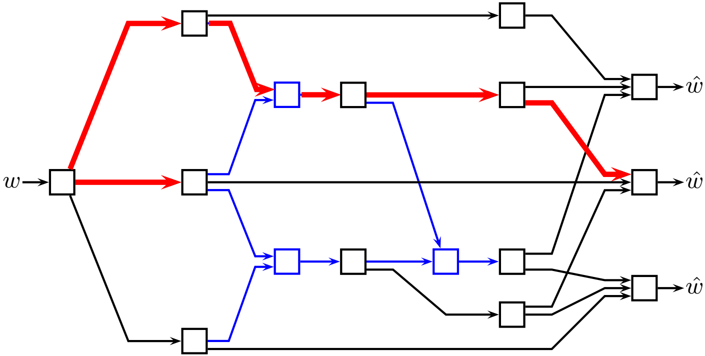

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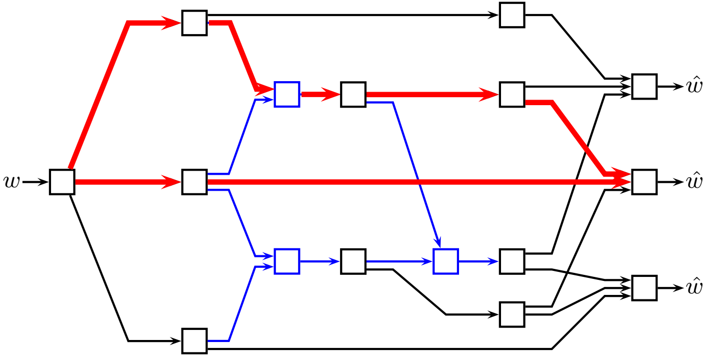

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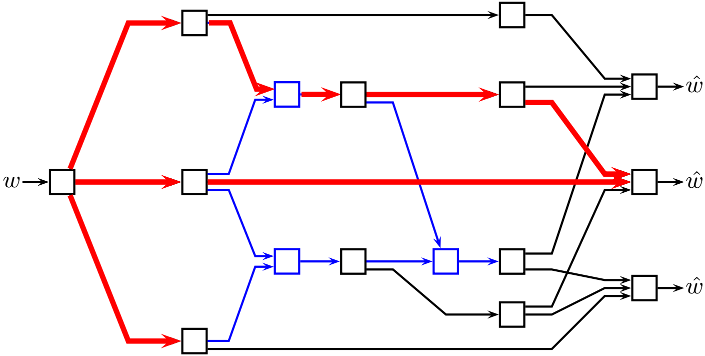

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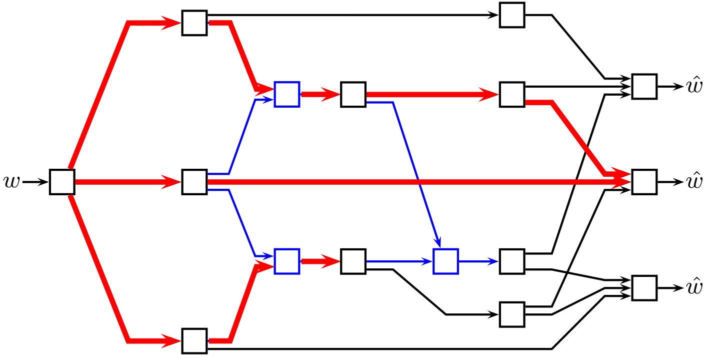

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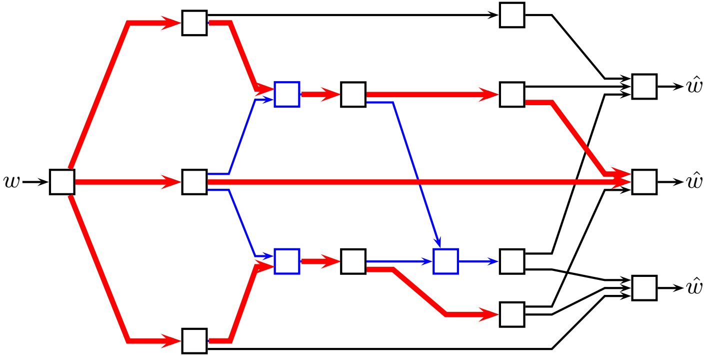

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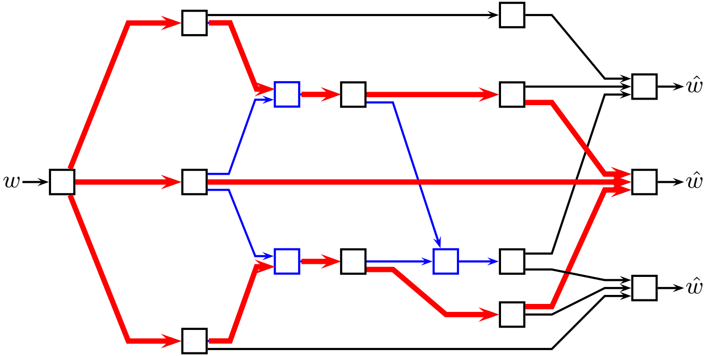

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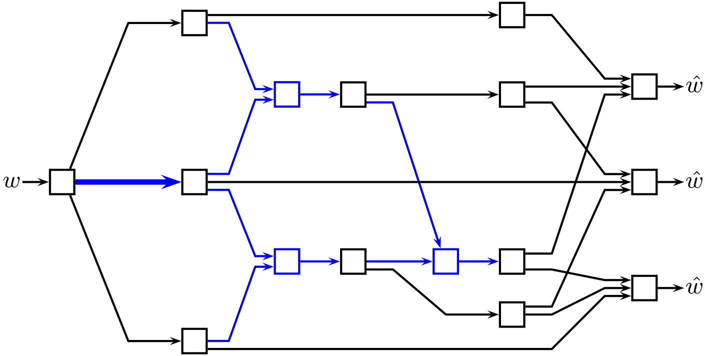

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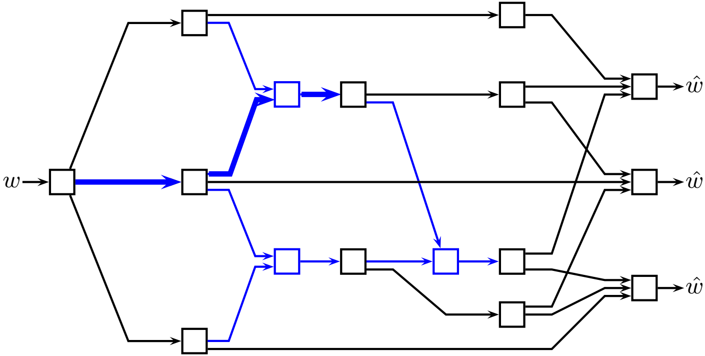

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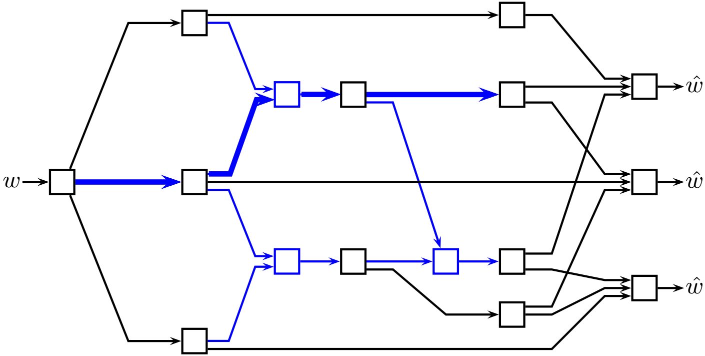

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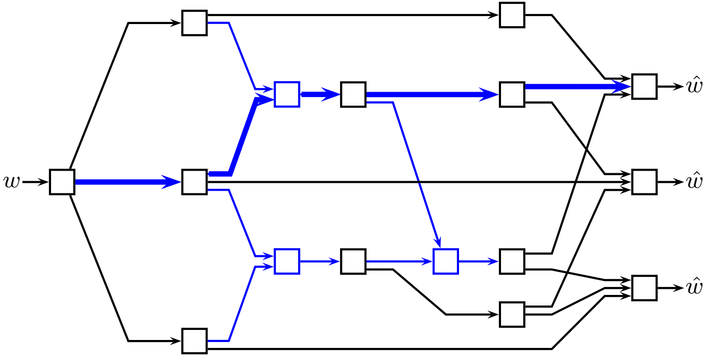

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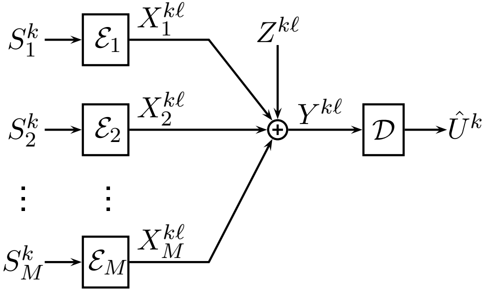
Proof Ideas

- *Convenient* to consider sending blocks of Gaussian sources at distortion targets and then treating these as supersymbols.
- Computing linear functions of Gaussians sources over Gaussian MACs and the associated linear processing rate.
- Map appropriate network code to our AWGN network with lattices.

From Gaussians to Bits

- Assume, for k large enough, we can send a length-k i.i.d. Gaussian source with variance σ^2 from our sender to every receiver with distortion D.
- Then we can multicast over the network at any rate less than $R(D) = \frac{1}{2}\log_2\left(\frac{\sigma^2}{D}\right)$.
- Proof Sketch: Fix encoding and decoding functions for every interior node in the network. Take Gaussian vectors as supersymbols in a new block code.

Linear Functions over a Gaussian MAC



- length-k Gaussian sources
- Want linear function $U = \alpha_1 S_1 + \alpha_2 S_2 + \cdots \alpha_M S_M$ at low distortion $D = E[(U - \hat{U})^2]$

- Erez-Litsyn-Zamir IT Trans. 2005: $\exists$ lattices good for both source and channel coding.
- Scale up each source and quantize onto the same lattice and transmit simultaneously. Receiver decodes the sum.
- Repeat ℓ times with encoders sending quantization errors.

Linear Processing Rate

- N. and Gastpar IT Trans. Oct. 2007: Linear function received at distortion at most:

$$D_\ell = \sigma_S^2 \max_j \alpha_j^2 \left(\frac{MN}{N+P} \right)^\ell$$

- Linear processing rate is given by $R_{LP} = \lim_{\ell \to \infty} \frac{1}{2\ell} \log_2 R(D_\ell)$
- Thus, structured random code can achieve at least

$$R_{LP} = \frac{1}{2} \log_2 \left(\frac{1}{M} + \frac{P}{N} \right)$$

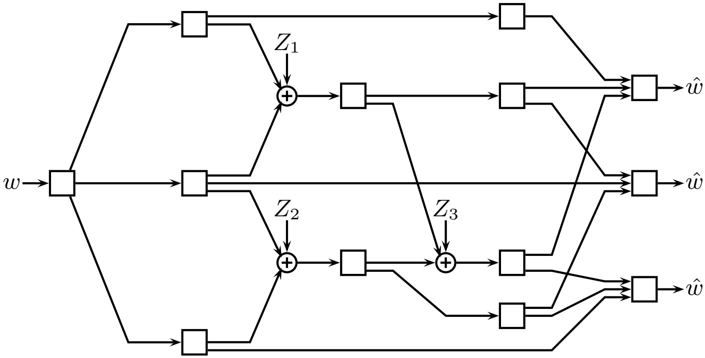
- IID random code only achieves:

$$R_{LP} = \frac{1}{2M} \log_2 \left(1 + \frac{MP}{N} \right)$$

Building a Network Code

- Reduction to point-to-point network:
  - Replacing all MACs with nodes with same connectivity
  - New node's link capacities are given by the MAC's linear processing rate
- Consider all channels in terms of equal capacity *chunks* and draw appropriate network code over prime-sized finite field.
- Network code equations also full rank over the reals.
- Refine Gaussian vectors across original network according to network code on the reals
- Receivers make LMMSE estimates.

Coding Theorem Restated



- Reduction to bit pipe network
- MACs become nodes

- New links capacities given by linear processing rate:

$$R_{LP} = \frac{1}{2}\log_2\left(\frac{1}{M_j} + \frac{P}{N_j}\right)$$

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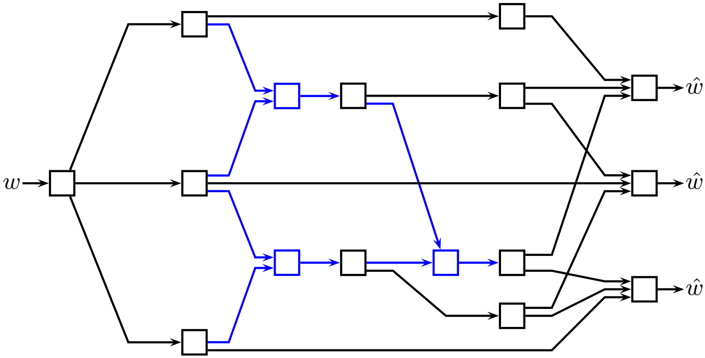

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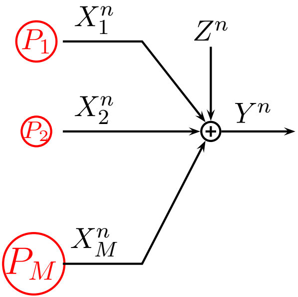
MAC with Unequal Users



- Different transmit power constraints:

$$\frac{1}{n}\sum_{i=1}^{n}(x_j[i])^2 \leq P_j$$

$$P_{AVG} = \frac{1}{M}\sum_{j=1}^{M} P_j$$

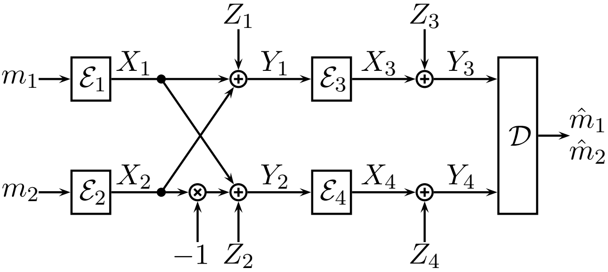
- Idea: Layer different linear functions on top of each other.

$$R_{LP,1} = \frac{1}{2}\log_2\left(\frac{1}{M} + \frac{\alpha P_{AVG}}{N}\right)$$

$$R_{LP,2} = \frac{1}{2}\log_2\left(\frac{1}{M} + \frac{(1-\alpha)P_{AVG}}{N + \alpha M P_{AVG}}\right)$$

Beyond Network Coding...

The "Sum-Difference" Relay MAC:



- Two senders, one receiver
- Equal transmit powers and noise variances throughout

The "Sum-Difference" Relay MAC.

Look at symmetric rate point, $R = R_1 = R_2$:

- Structured random code allows one relay to decode the sum and the other the difference:

$$R = \frac{1}{2}\log_2\left(\frac{1}{2} + \frac{P}{N}\right).$$

- IID random code results in decoding at the relays or compress-and-forward:

$$R_{DF} = \frac{1}{4}\log_2\left(1 + \frac{2P}{N}\right)$$
$$R_{CF} = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\left(\frac{2P}{3P+N}\right)\right).$$

Concluding Remarks

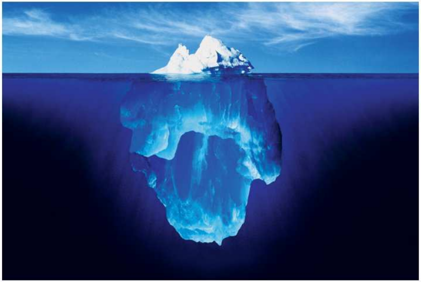
Lattices can be extremely useful for solving AWGN communication problems.

This has been observed by others, too:

- Philosof-Khisti-Erez-Zamir ISIT 2007: Lattices help for MAC with two interferences, one known at each encoder
- Krithivasan-Pradhan arXiv July 2007: Lattices help for distributed source coding of difference of correlated Gaussians
- Narayanan-Wilson-Sprintson Allerton 2007: Lattices help for two-way relaying
- Bresler-Parekh-Tse Allerton 2007: Lattices help on a many-to-one Gaussian interference channel

Concluding Remarks

Structured random codes will be required to prove capacity (and rate-distortion) results for many networks to come...



Some previous work:

- N. and Gastpar IT Trans. October 2007: Computation over Multiple-Access Channels
- N. and Gastpar ITW 2007 Lake Tahoe: The Case for Structured Random Codes in Network Communication Theorems