Ergodic Interference Alignment

Bobak Nazer* , Michael Gastpar*,
Syed Ali Jafar†, and Sriram Vishwanath‡

*EECS Department, UC Berkeley
†EECS Department, UC Irvine
‡ECE Department, UT Austin

ISIT 2009

July 2, 2009

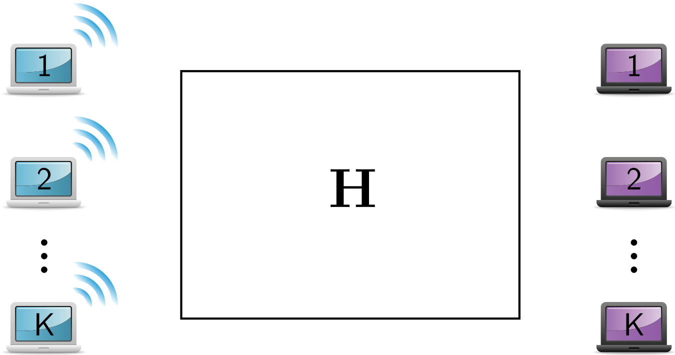
K-User Interference Channel

- K transmitter-receiver pairs share a common wireless channel.

K-User Interference Channel

- K transmitter-receiver pairs share a common wireless channel.
- Receivers observe noisy linear combinations of transmitted signals:

K-User Interference Channel



- K transmitter-receiver pairs share a common wireless channel.
- Receivers observe noisy linear combinations of transmitted signals:

$$Y_k = \sum_{\ell=1}^{K} h_{k\ell} X_\ell + Z_k$$

K-User Interference Channel

$$\begin{array}{cccc} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{array}$$

- K transmitter-receiver pairs share a common wireless channel.
- Receivers observe noisy linear combinations of transmitted signals:

$$Y_k = \sum_{\ell=1}^{K} h_{k\ell} X_\ell + Z_k$$

Strong Interference

Strong Interference

Strong Interference

- Strategy: decode and remove interfering signals.

Strong Interference

- Strategy: decode and remove interfering signals.
- Optimal if interference is very strong.
  (**Carleial '75, Sato '81, Han-Kobayashi '81, Sankar-Erkip-Poor '08, Sridharan-Jafarian-Jafar-Vishwanath '08**)

Weak Interference

- Strategy: treat interference as noise.

Weak Interference

- Strategy: treat interference as noise.
- Optimal if interference is very weak.
  (**Motahari-Khandani '07, Shang-Kramer-Chen '07, Annapureddy-Veeravalli '08**)

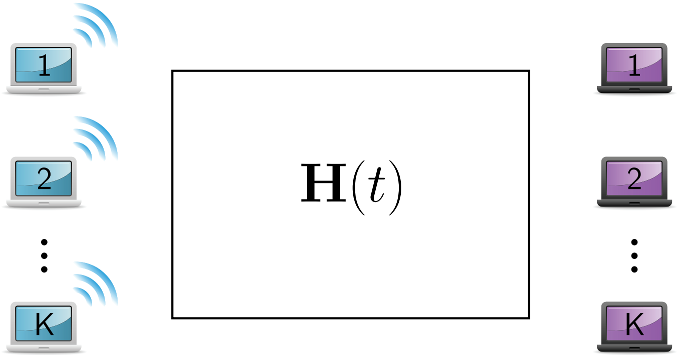
Weak Interference

- Strategy: treat interference as noise.
- Optimal if interference is very weak.
  (**Motahari-Khandani '07, Shang-Kramer-Chen '07, Annapureddy-Veeravalli '08**)

Weak Interference

- Strategy: treat interference as noise.
- Optimal if interference is very weak.
  (**Motahari-Khandani '07, Shang-Kramer-Chen '07, Annapureddy-Veeravalli '08**)

K-User Fast Fading Interference Channel



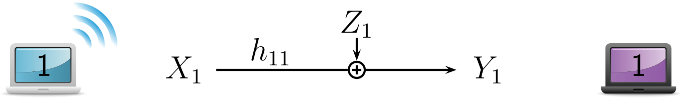
- Channel coefficients have i.i.d. uniform phase.
- Transmitters and receivers know $\mathbf{H}(t)$ causally.
- Usual power constraint P_k.
- Noise i.i.d. $\mathcal{CN}(0,1)$.

K-User Fast Fading Interference Channel

$$\begin{bmatrix} h_{11}(t) & h_{12}(t) & \cdots & h_{1K}(t) \\ h_{21}(t) & h_{22}(t) & \cdots & h_{2K}(t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1}(t) & h_{K2}(t) & \cdots & h_{KK}(t) \end{bmatrix}$$

- Channel coefficients have i.i.d. uniform phase.
- Transmitters and receivers know $\mathbf{H}(t)$ causally.
- Usual power constraint P_k.
- Noise i.i.d. $\mathcal{CN}(0,1)$.

Interference-Free Capacity



- Interference-free capacity:

$$R_k^{\mathsf{FREE}} = E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]$$

Interference-Free Capacity

- Interference-free capacity:

$$R_k^{\mathsf{FREE}} = E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]$$

Time Division

- Eliminate interference through time division:

$$R_k^{\text{TDMA}} = \frac{1}{K} E_{\mathbf{H}} \left[\log \left(1 + K |h_{kk}|^2 P_k \right) \right]$$

Time Division

- Eliminate interference through time division:

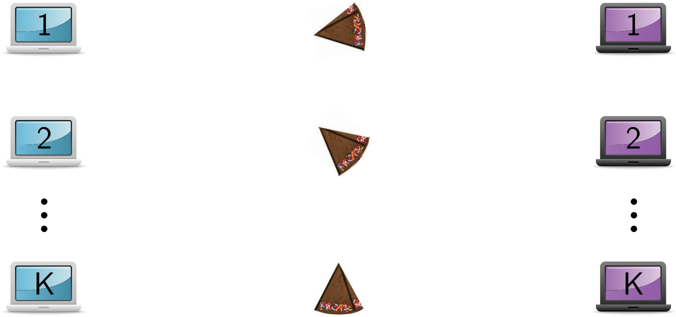
$$R_k^{\text{TDMA}} = \frac{1}{K} E_{\mathbf{H}} \left[\log \left(1 + K |h_{kk}|^2 P_k \right) \right]$$

Time Division

- Eliminate interference through time division:

$$R_k^{\mathsf{TDMA}} = \frac{1}{K} E_{\mathbf{H}} \left[\log \left(1 + K |h_{kk}|^2 P_k \right) \right]$$

Time Division



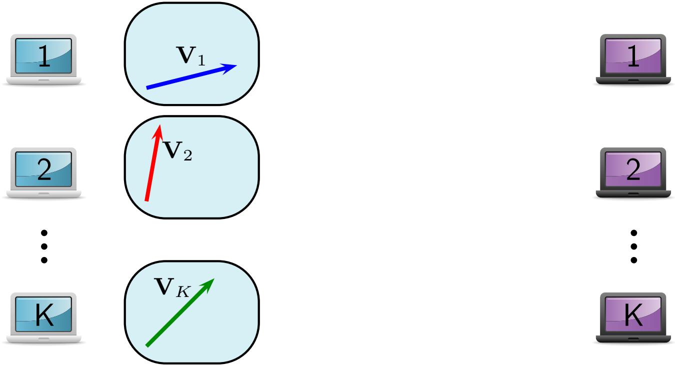
- Eliminate interference through time division:

$$R_k^{\text{TDMA}} = \frac{1}{K} E_{\mathbf{H}} \left[\log \left(1 + K |h_{kk}|^2 P_k \right) \right]$$

Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\to \infty$:

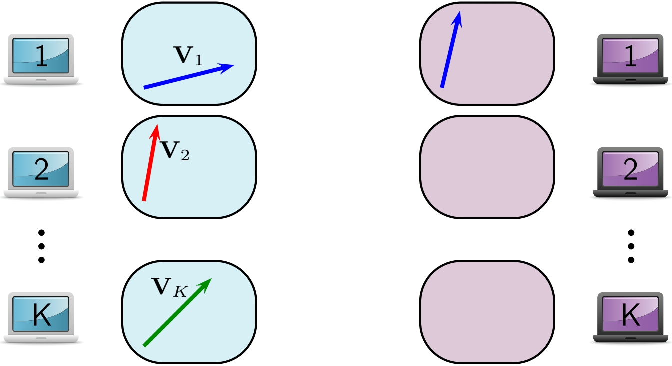
$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]} = \frac{1}{2}$$

Interference Alignment



Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\rightarrow \infty$:

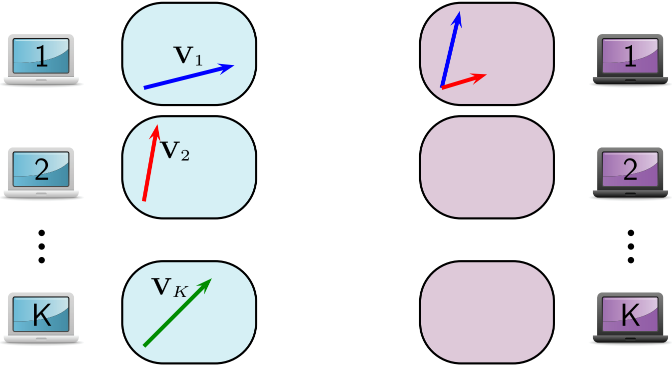
$$\lim_{P_k \rightarrow \infty} \frac{R_k^{IA}}{E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]} = \frac{1}{2}$$



Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\to \infty$:

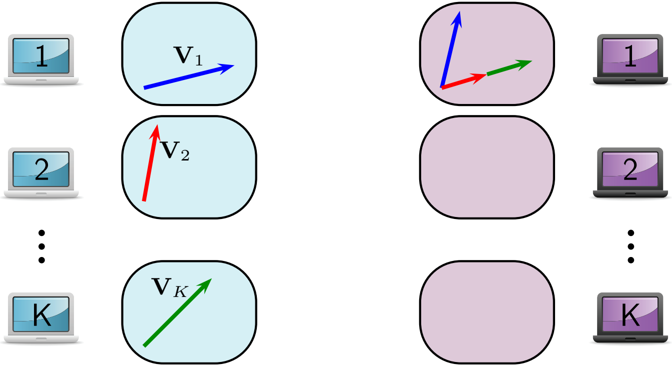
$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]} = \frac{1}{2}$$

Interference Alignment



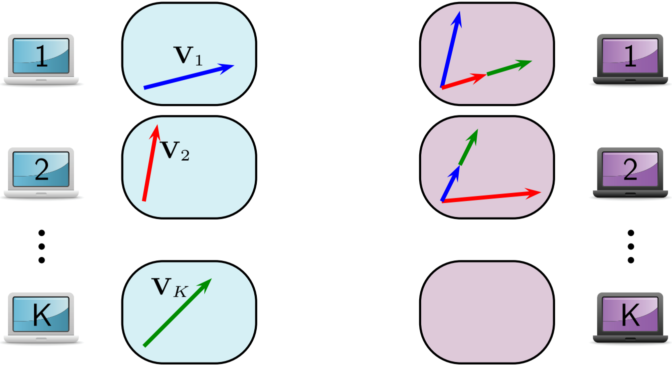
Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\to \infty$:

$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]} = \frac{1}{2}$$



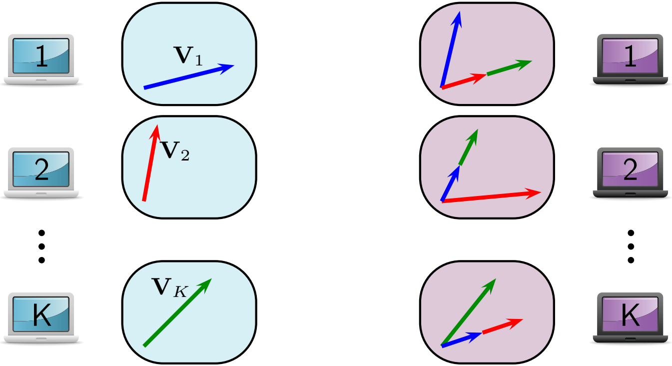
Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\to \infty$:

$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}} \left[\log \left(1 + |h_{kk}|^2 P_k \right) \right]} = \frac{1}{2}$$



Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\to \infty$:

$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}}\left[\log\left(1 + |h_{kk}|^2 P_k\right)\right]} = \frac{1}{2}$$



Cadambe-Jafar '08: With careful choice of precoding matrices, each user can get "half the cake" as the SNR $\rightarrow \infty$:

$$\lim_{P_k \to \infty} \frac{R_k^{IA}}{E_{\mathbf{H}} \left[\log \left(1 + |h_{kk}|^2 P_k \right) \right]} = \frac{1}{2}$$

- **New Scheme**: gets (slightly more than) half the interference-free rate at any SNR!

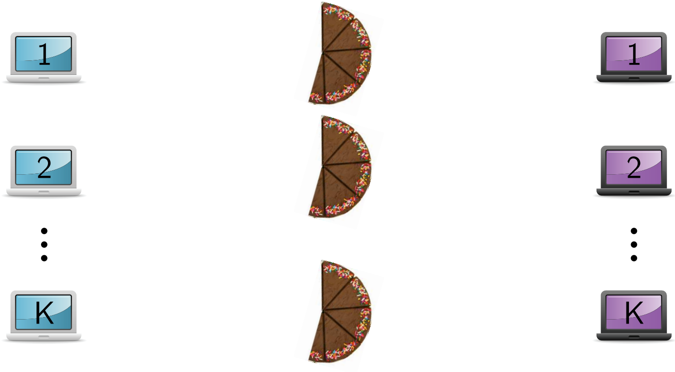
$$R_k^{\mathsf{EIA}} = \frac{1}{2} E_{\mathbf{H}} \left[\log \left(1 + 2|h_{kk}|^2 P_k \right) \right]$$

Ergodic Interference Alignment

- **New Scheme**: gets (slightly more than) half the interference-free rate at any SNR!

$$R_k^{\text{EIA}} = \frac{1}{2} E_{\mathbf{H}} \left[\log \left(1 + 2|h_{kk}|^2 P_k \right) \right]$$

Ergodic Interference Alignment



- **New Scheme**: gets (slightly more than) half the interference-free rate at any SNR!

$$R_k^{\mathsf{EIA}} = \frac{1}{2} E_{\mathbf{H}} \left[\log \left(1 + 2|h_{kk}|^2 P_k \right) \right]$$

Key Idea

1. At time t with channel $\mathbf{H}$, user k transmits signal X_k.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix}$$

Key Idea

1. At time t with channel $\mathbf{H}$, user k transmits signal X_k.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix}$$

2. When complementary matrix $\mathbf{H}_C$ occurs, retransmit signals X_k.

$$\mathbf{H}_C = \begin{bmatrix} h_{11} & -h_{12} & \cdots & -h_{1K} \\ -h_{21} & h_{22} & \cdots & -h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{K1} & -h_{K2} & \cdots & h_{KK} \end{bmatrix}$$

Key Idea

1. At time t with channel $\mathbf{H}$, user k transmits signal X_k.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix}$$

2. When complementary matrix $\mathbf{H}_C$ occurs, retransmit signals X_k.

$$\mathbf{H}_C = \begin{bmatrix} h_{11} & -h_{12} & \cdots & -h_{1K} \\ -h_{21} & h_{22} & \cdots & -h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{K1} & -h_{K2} & \cdots & h_{KK} \end{bmatrix} \pm \delta$$

Key Idea

1. At time t with channel $\mathbf{H}$, user k transmits signal X_k.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix}$$

2. When complementary matrix $\mathbf{H}_C$ occurs, retransmit signals X_k.

$$\mathbf{H}_C = \begin{bmatrix} h_{11} & -h_{12} & \cdots & -h_{1K} \\ -h_{21} & h_{22} & \cdots & -h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{K1} & -h_{K2} & \cdots & h_{KK} \end{bmatrix} \pm \delta$$

3. Otherwise, transmit new signals and wait for their $\mathbf{H}_C$.

Ergodic Alignment at the Receivers

$$\begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_K(t) \end{bmatrix}$$

$$\begin{bmatrix} Y_1(t_C) \\ Y_2(t_C) \\ \vdots \\ Y_K(t_C) \end{bmatrix}$$

Ergodic Alignment at the Receivers

$$\begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_K(t) \end{bmatrix}$$

$$+ \quad \begin{bmatrix} Y_1(t_C) \\ Y_2(t_C) \\ \vdots \\ Y_K(t_C) \end{bmatrix}$$

Ergodic Alignment at the Receivers

$$\begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_K(t) \end{bmatrix}$$

$$+ \quad \begin{bmatrix} Y_1(t_C) \\ Y_2(t_C) \\ \vdots \\ Y_K(t_C) \end{bmatrix}$$

$$\begin{bmatrix} Y_1(t) + Y_1(t_C) \\ Y_2(t) + Y_2(t_C) \\ \vdots \\ Y_K(t) + Y_K(t_C \end{bmatrix}$$

Ergodic Alignment at the Receivers

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix} \quad \mathbf{X} \quad + \quad \mathbf{Z}(t)$$

$$+ \quad \left(\begin{bmatrix} h_{11} & -h_{12} & \cdots & -h_{1K} \\ -h_{21} & h_{22} & \cdots & -h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{K1} & -h_{K2} & \cdots & h_{KK} \end{bmatrix} \pm \delta \right) \mathbf{X} \quad + \quad \mathbf{Z}(t_C)$$

$$\begin{bmatrix} Y_1(t) + Y_1(t_C) \\ Y_2(t) + Y_2(t_C) \\ \vdots \\ Y_K(t) + Y_K(t_C \end{bmatrix}$$

Ergodic Alignment at the Receivers

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1K} \\ h_{21} & h_{22} & \cdots & h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KK} \end{bmatrix} \quad \mathbf{X} \quad + \quad \mathbf{Z}(t)$$

$$+ \quad \left(\begin{bmatrix} h_{11} & -h_{12} & \cdots & -h_{1K} \\ -h_{21} & h_{22} & \cdots & -h_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{K1} & -h_{K2} & \cdots & h_{KK} \end{bmatrix} \pm \delta \right) \mathbf{X} \quad + \quad \mathbf{Z}(t_C)$$

$$\left(\begin{bmatrix} 2h_{11} & 0 & \cdots & 0 \\ 0 & 2h_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2h_{KK} \end{bmatrix} \pm \delta \right) \mathbf{X} \quad + \quad \mathbf{Z}(t) + \mathbf{Z}(t_C)$$

Ergodic Interference Alignment

Sum of channel observations is (nearly) interference-free:

$$\mathbf{H} + \mathbf{H}_C = \begin{bmatrix} 2h_{11} & & 0 \\ & \ddots & \\ 0 & & 2h_{MM} \end{bmatrix} \pm \delta$$

Worst case SINR:

$$2P_k \left(\frac{|h_{kk}|^2 - 2\delta(Re(h_{kk}) + Im(h_{kk})) + \delta^2}{1 + 4\delta^2 \sum_{\ell \neq k} P_\ell} \right)$$

Ergodic Interference Alignment

Sum of channel observations is (nearly) interference-free:

$$\mathbf{H} + \mathbf{H}_C = \begin{bmatrix} 2h_{11} & & 0 \\ & \ddots & \\ 0 & & 2h_{MM} \end{bmatrix} \pm \delta$$

Worst case SINR:

$$\lim_{\delta \downarrow 0} \; 2P_k \left(\frac{|h_{kk}|^2 - 2\delta(Re(h_{kk}) + Im(h_{kk})) + \delta^2}{1 + 4\delta^2 \sum_{\ell \neq k} P_\ell} \right) = 2|h_{kk}|^2 P_k$$

Pairing Up Channels

- We need to match up almost every matrix with its complement.

Pairing Up Channels

- We need to match up almost every matrix with its complement.
- Want a finite set of possible matrices $\mathcal{H}$ for analysis:

Pairing Up Channels

- We need to match up almost every matrix with its complement.
- Want a finite set of possible matrices $\mathcal{H}$ for analysis:

    1. Quantize each channel coefficient to precision δ (closest point in $\delta(\mathbb{Z} + j\mathbb{Z})$).

Pairing Up Channels

- We need to match up almost every matrix with its complement.
- Want a finite set of possible matrices $\mathcal{H}$ for analysis:
  1. Quantize each channel coefficient to precision δ (closest point in $\delta(\mathbb{Z} + j\mathbb{Z})$).
  2. Set threshold h_{MAX}. Throw out any matrix with $|h_{k\ell}| > h_{\mathsf{MAX}}$.

Pairing Up Channels

- We need to match up almost every matrix with its complement.
- Want a finite set of possible matrices $\mathcal{H}$ for analysis:
  1. Quantize each channel coefficient to precision δ (closest point in $\delta(\mathbb{Z} + j\mathbb{Z})$).
  2. Set threshold h_{MAX}. Throw out any matrix with $|h_{k\ell}| > h_{\mathsf{MAX}}$.
- Choose δ, h_{MAX} to get desired rate gap.

Pairing Up Channels

- We need to match up almost every matrix with its complement.
- Want a finite set of possible matrices $\mathcal{H}$ for analysis:
  1. Quantize each channel coefficient to precision δ (closest point in $\delta(\mathbb{Z} + j\mathbb{Z})$).
  2. Set threshold h_{MAX}. Throw out any matrix with $|h_{k\ell}| > h_{\text{MAX}}$.
- Choose δ, h_{MAX} to get desired rate gap.
- Since phase is i.i.d. uniform, $P(\mathbf{H}) = P(\mathbf{H}_C)$.

Convergence in Type

Sequence of channel matrices $\mathbf{H}^n$ is ϵ-typical if:

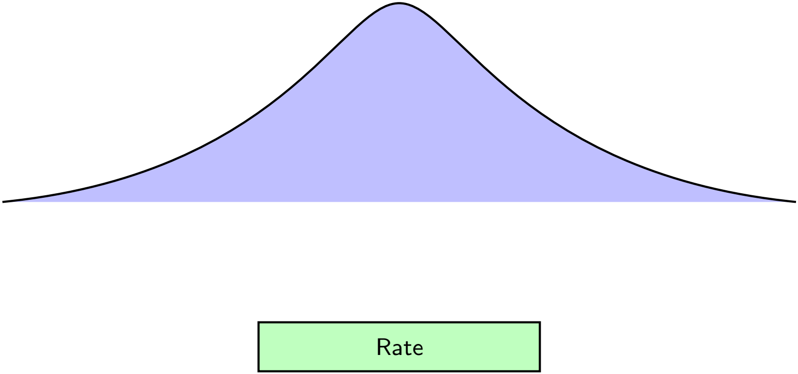
$$\left| \frac{1}{n} N(\mathsf{H} | \mathbf{H}^n) - P(\mathsf{H}) \right| \leq \epsilon \quad \forall \mathsf{H} \in \mathcal{H}$$

Lemma (Csiszar-Körner 2.12)

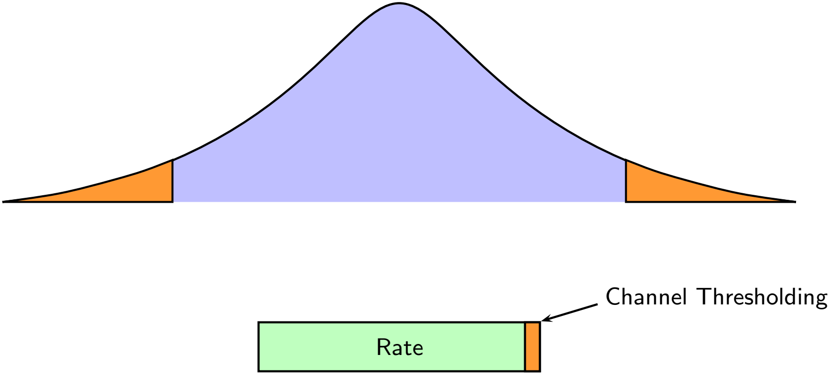
For any i.i.d. sequence, $\mathbf{H}^n$, the probability of the set of all ϵ-typical sequences, A_ϵ^n, is lower bounded by:

$$P(A_\epsilon^n) \geq 1 - \frac{|\mathcal{H}|}{4n\epsilon^2}$$

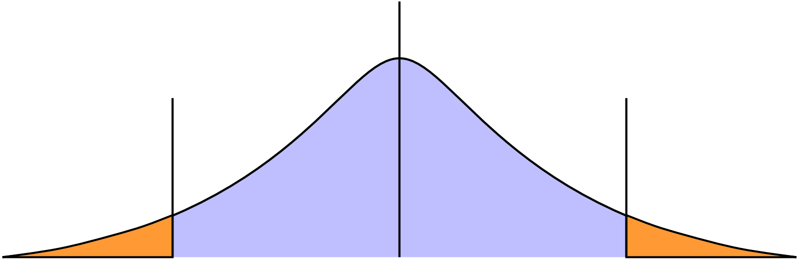
Convergence in Type

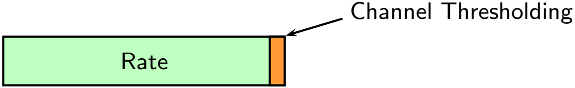


Convergence in Type

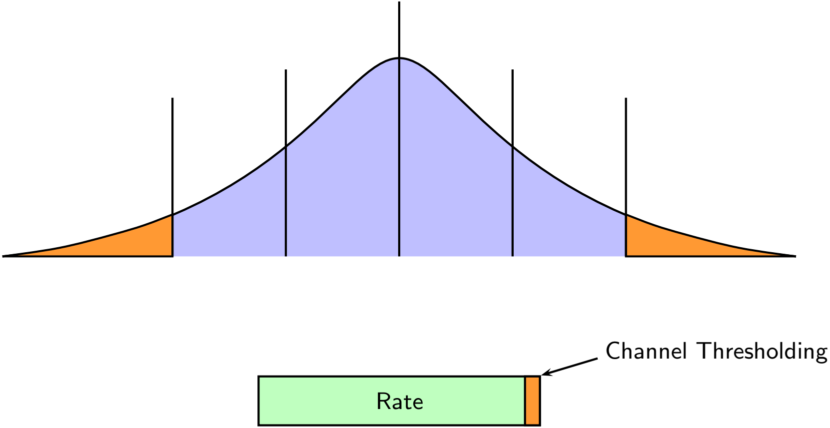


Convergence in Type

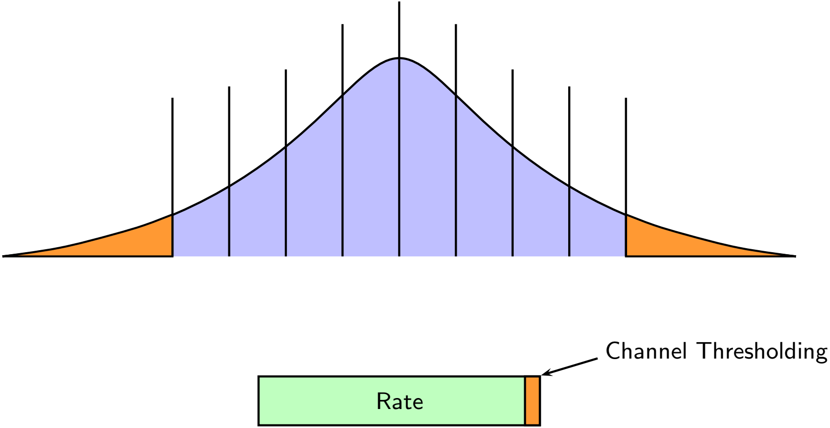




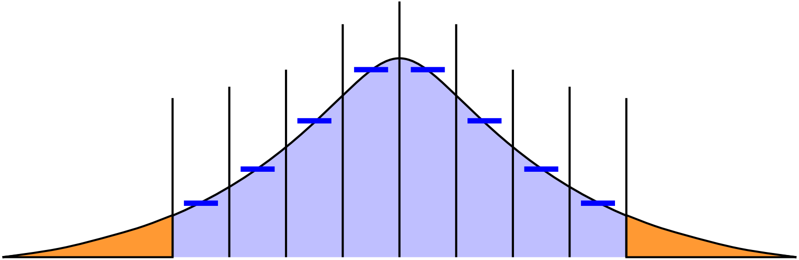
Convergence in Type

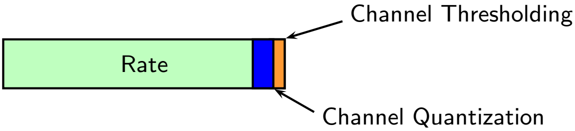


Convergence in Type

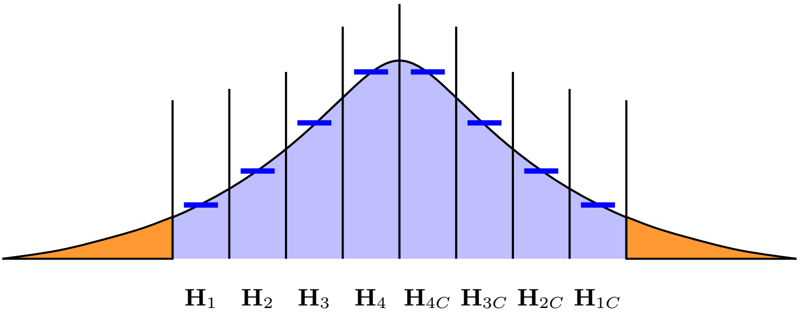


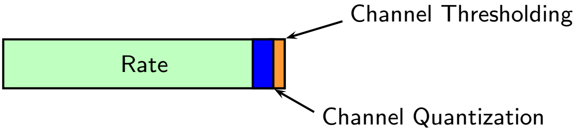
Convergence in Type



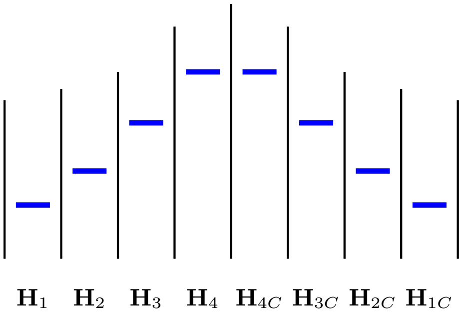


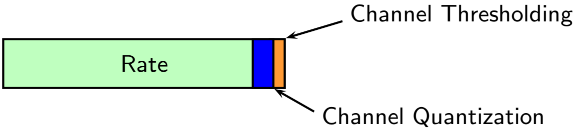
Convergence in Type



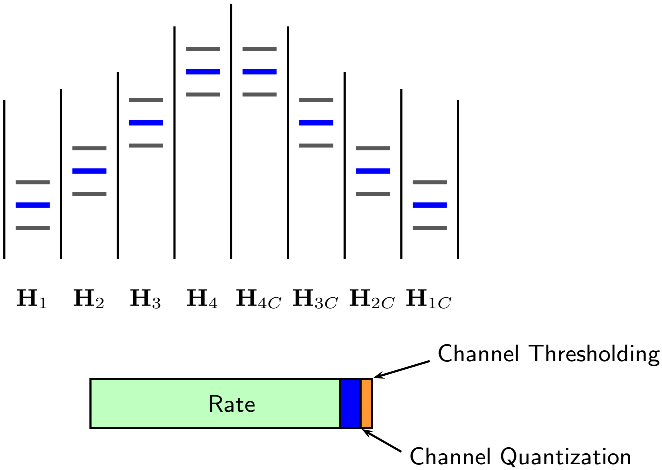


Convergence in Type

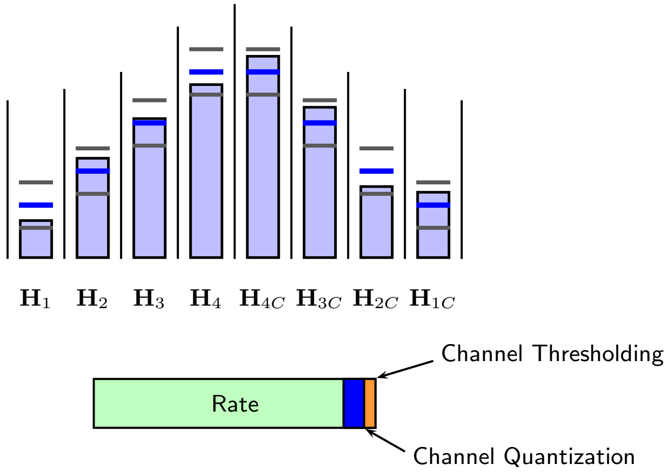




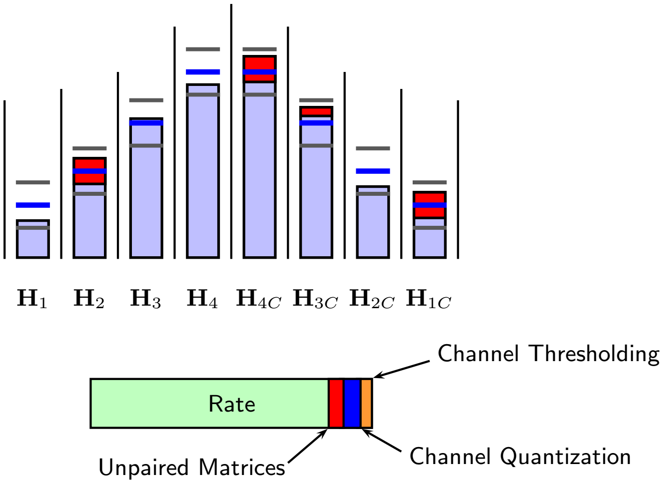
Convergence in Type



Convergence in Type



Convergence in Type



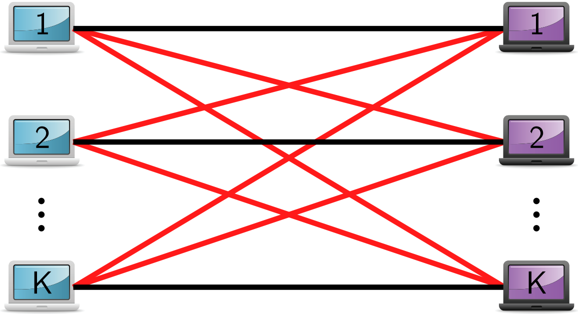
Achievable Rate

Theorem

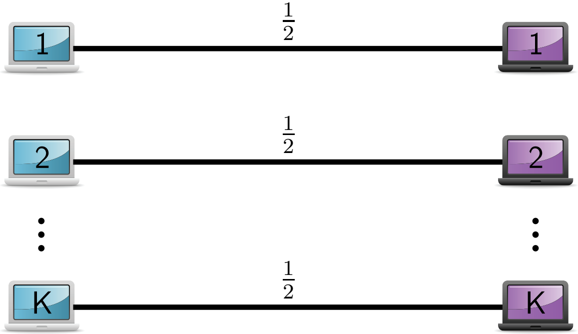
Each user can achieve at least half its interference-free capacity at **any signal-to-noise ratio***:*

$$R_k = \frac{1}{2} E \left[\log \left(1 + 2|h_{kk}|^2 P_k \right) \right] > \frac{1}{2} R_k^{\mathit{FREE}}$$

Network Transformation

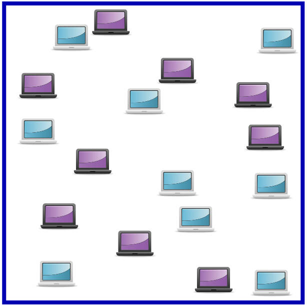


Network Transformation



Phase Fading

- **Jafar '09:** Whenever the channel is in a bottleneck state, ergodic alignment achieves the capacity.
- Example: K transmitter-receiver pairs randomly placed in a square. Signal strength governed by distance. As $K \to \infty$, ergodic alignment achieves capacity.



Rayleigh Fading

- For Rayleigh fading, we get a very weak interference channel with some constant probability $\rho > 0$.

Rayleigh Fading

- For Rayleigh fading, we get a very weak interference channel with some constant probability $\rho > 0$.
- Ignore all interference in weak interference case. Get R_k^{WEAK}.

Rayleigh Fading

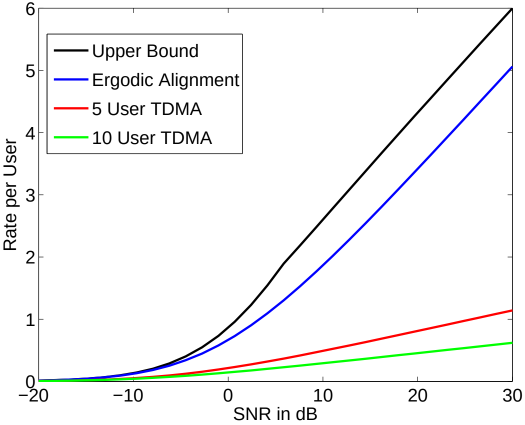
- For Rayleigh fading, we get a very weak interference channel with some constant probability $\rho > 0$.
- Ignore all interference in weak interference case. Get R_k^{WEAK}.
- Otherwise, use ergodic alignment to get R_k^{EA}.

Rayleigh Fading

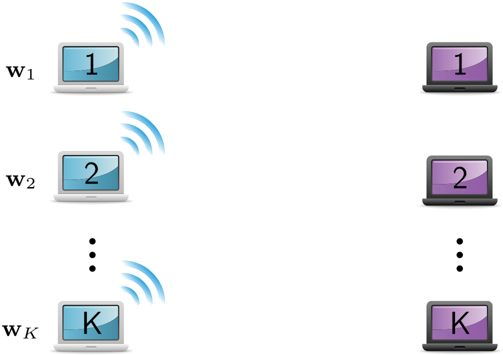
- For Rayleigh fading, we get a very weak interference channel with some constant probability $\rho > 0$.
- Ignore all interference in weak interference case. Get R_k^{WEAK}.
- Otherwise, use ergodic alignment to get R_k^{EA}.
- Each user gets $R_k = \rho R_k^{\mathsf{WEAK}} + (1-\rho) R_k^{\mathsf{EA}} > R_k^{\mathsf{EA}}$

Rayleigh Fading

- Channel coefficients i.i.d. Rayleigh.
- Equal transmit power per user.

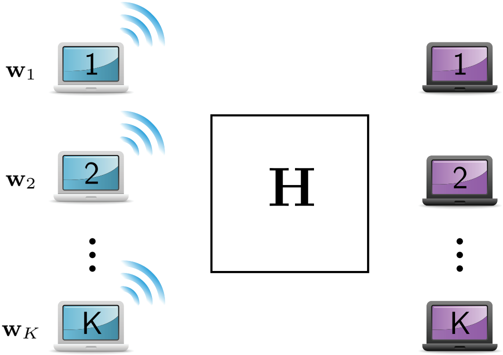


Finite Field Interference Channels



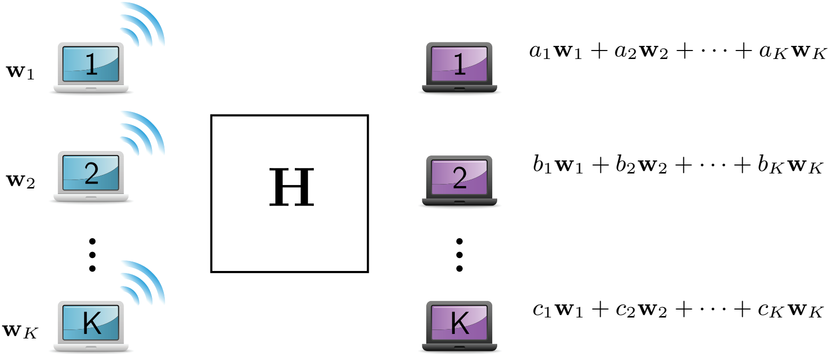
- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t) X_\ell(t) + Z_k(t)$

Finite Field Interference Channels



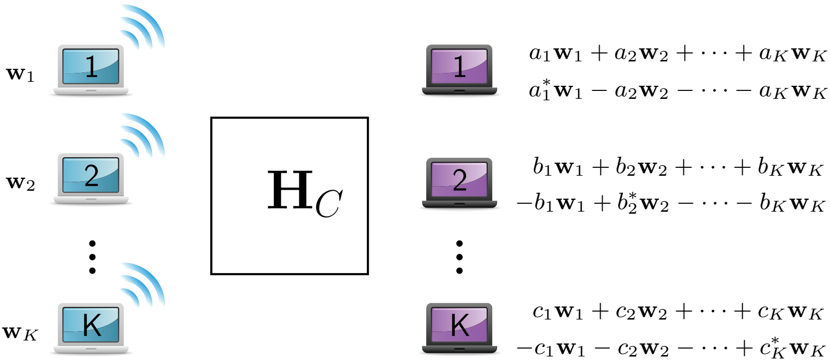
- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t) X_\ell(t) + Z_k(t)$
- We need to decode a function to eliminate noise.

Finite Field Interference Channels



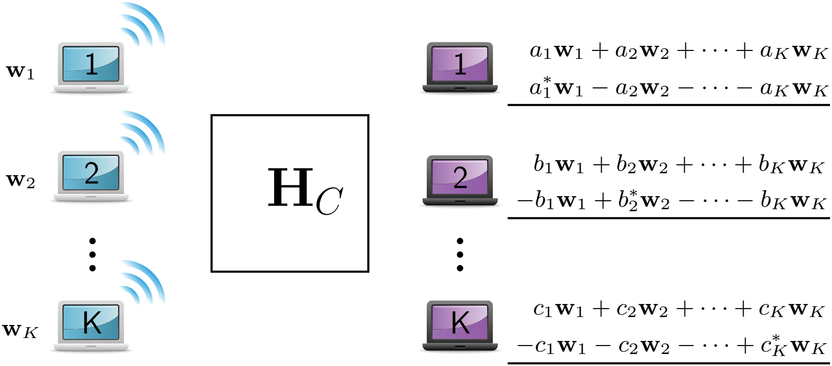
- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t)X_\ell(t) + Z_k(t)$
- We need to decode a function to eliminate noise.

Finite Field Interference Channels



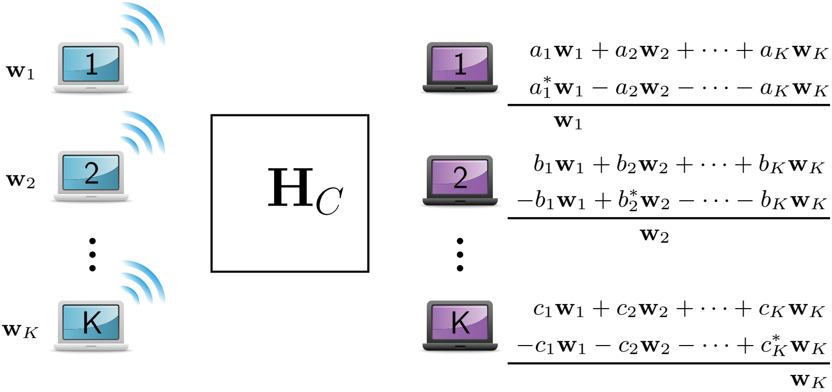
- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t) X_\ell(t) + Z_k(t)$
- We need to decode a function to eliminate noise.

Finite Field Interference Channels



- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t)X_\ell(t) + Z_k(t)$
- We need to decode a function to eliminate noise.

Finite Field Interference Channels



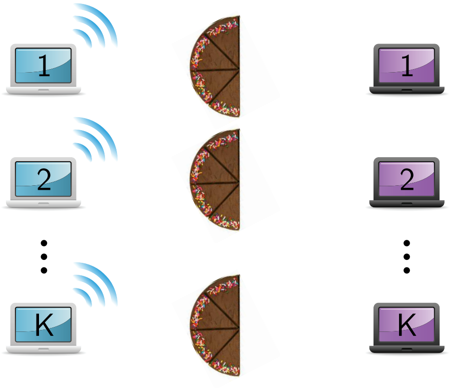
- Finite field channels: $Y_k(t) = \sum_{\ell=1}^{K} h_{k\ell}(t) X_\ell(t) + Z_k(t)$
- We need to decode a function to eliminate noise.

Finite Field IC: Ergodic Capacity Region

- Use computation codes from **Nazer-Gastpar '07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

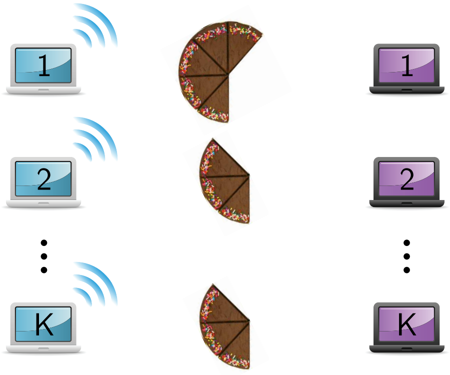
Finite Field IC: Ergodic Capacity Region



- Use computation codes from **Nazer-Gastpar '07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

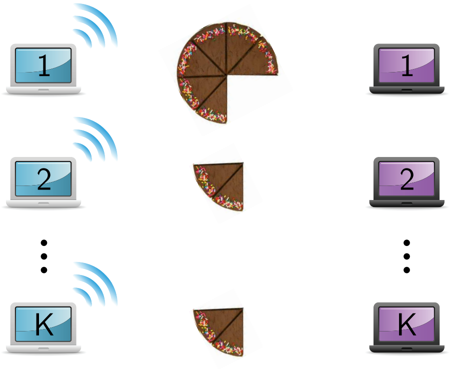
Finite Field IC: Ergodic Capacity Region



- Use computation codes from **Nazer-Gastpar '07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

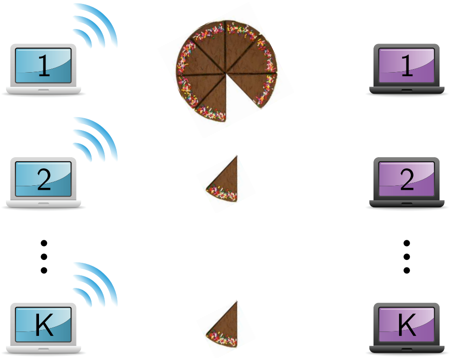
Finite Field IC: Ergodic Capacity Region



- Use computation codes from **Nazer-Gastpar '07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

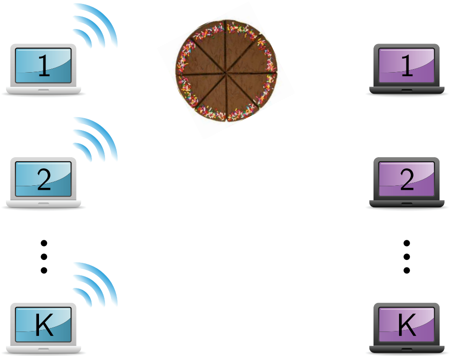
Finite Field IC: Ergodic Capacity Region



- Use computation codes from **Nazer-Gastpar '07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

Finite Field IC: Ergodic Capacity Region



- Use computation codes from **Nazer-Gastpar ’07**.
- Ergodic capacity region for K-user finite field interference channel:

$$R_\ell + R_k \leq \log_2 q - H(Z), \qquad \forall k \neq \ell.$$

Related Work

Alignment over Finite Field Multi-Hop Relay Networks (**Jeon-Chung '09**)

Interference Alignment for MIMO X Channels
(**Maddah-Ali - Motahari - Khandani '08**)

Inseparability of Parallel Interference Channels (**Cadambe-Jafar '08, Sankar-Shang-Erkip-Poor '08**)

Structured Codes for Interference Channels (**Bresler-Parekh-Tse '07, Sridharan-Jafarian-Vishwanath-Jafar-Shamai '08**)

Conclusions

- Developed a new interference alignment scheme that allows each user to attain half its interference-free capacity at any SNR.
- For certain channel models, showed that ergodic interference alignment achieves the capacity.