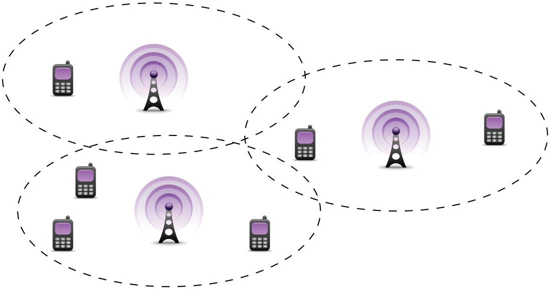
Integer-Forcing for Cloud Radios

Islam El Bakoury and Bobak Nazer

Boston University

ITA 2020

Cloud Radio Access Networks (C-RAN)



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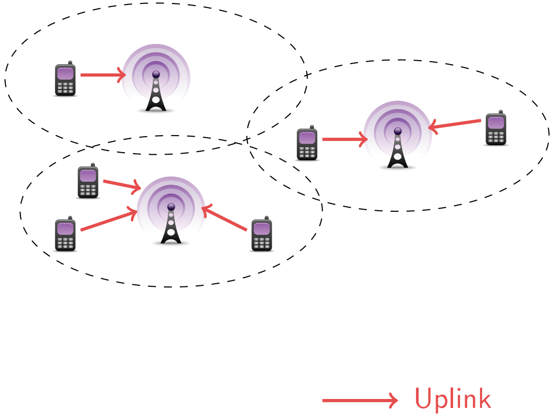

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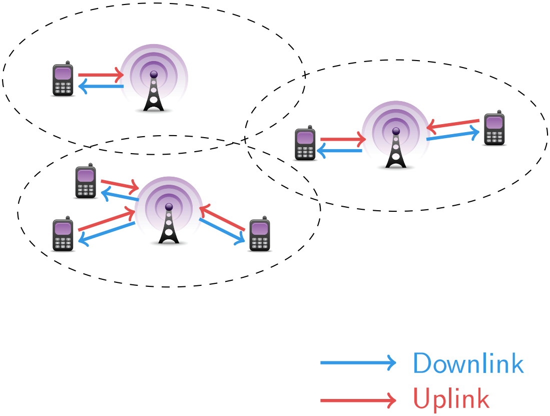

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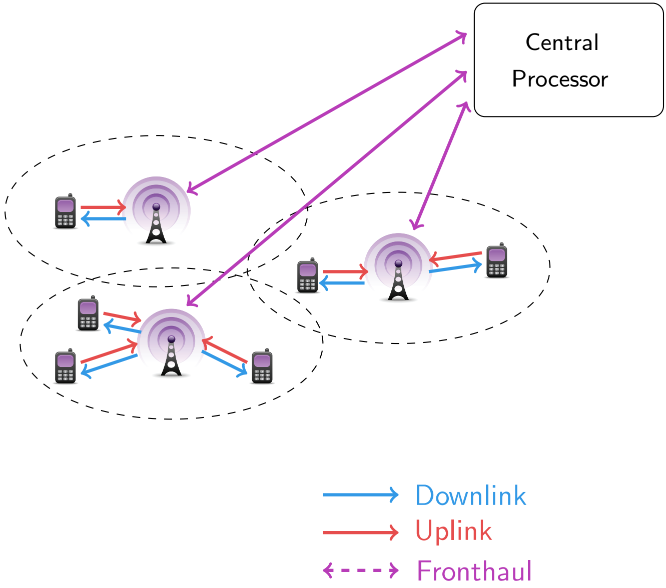
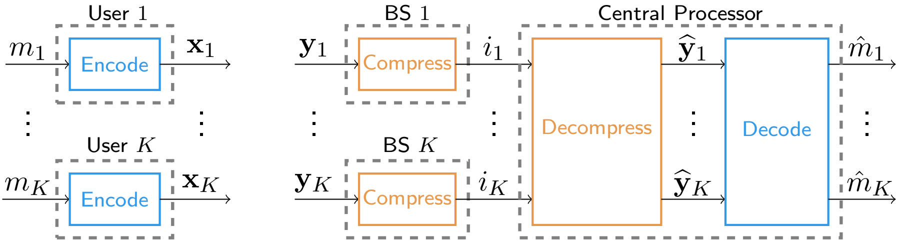


Uplink


User 1
m_1
Encode
$\mathbf{x}_1$
User K
m_K
Encode
$\mathbf{x}_K$
BS 1
$\mathbf{y}_1$
Compress
i_1
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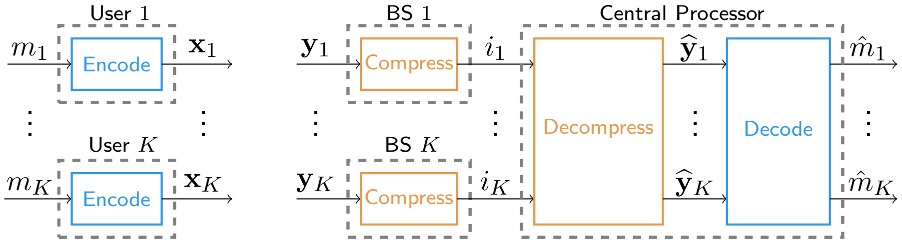
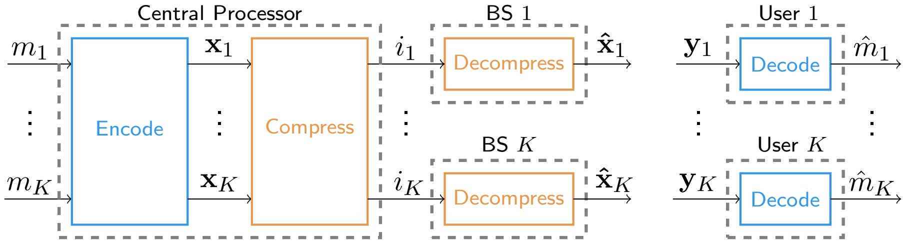


Downlink



Uplink Channel $\equiv$ MIMO MAC

- Channel Model: $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$.
- Joint Typicality: Search for codewords $\tilde{\mathbf{X}}$ with $(\tilde{\mathbf{X}}, \mathbf{Y})$ jointly typical.
- Successive Cancellation: $\tilde{\mathbf{y}}_k = \mathbf{Y} - \sum_{i=1}^{k-1} \mathbf{h}_i \mathbf{x}_i^\mathsf{T} = \sum_{i=k}^{K} \mathbf{h}_i \mathbf{x}_i^\mathsf{T} + \mathbf{Z}$
- Integer-Forcing: $\mathbf{B}\mathbf{Y} = \mathbf{B}\mathbf{H}\mathbf{X} + \mathbf{B}\mathbf{Z} = \mathbf{A}\mathbf{X} + \underbrace{(\mathbf{B}\mathbf{H} - \mathbf{A})\mathbf{X} + \mathbf{B}\mathbf{Z}}_{\text{effective noise}}$

$\mathbf{A} \in \mathbb{Z}^{K \times K}$ is full rank and $\mathbf{B}$ is MMSE matrix.

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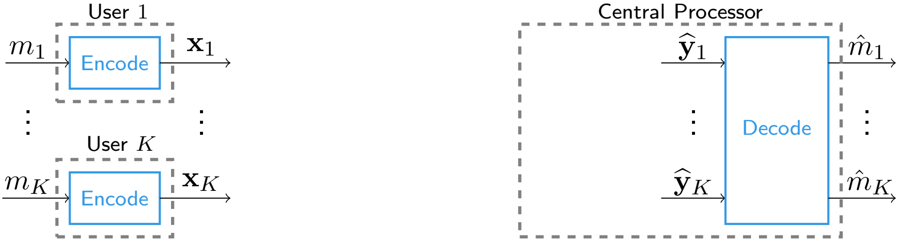

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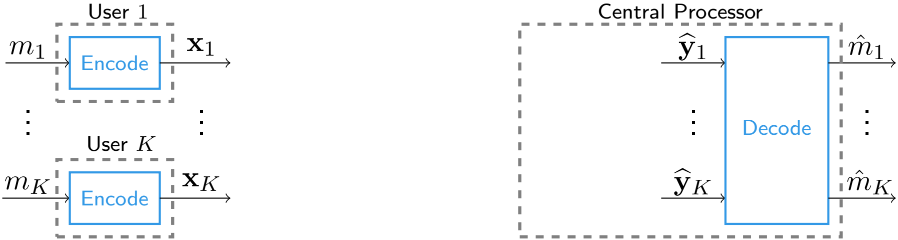

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- Channel Model: $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$.
- Dirty Paper Encoding: Encode $\mathbf{x}_i$ treating $(\mathbf{x}_1, \ldots, \mathbf{x}_{i-1})$ as dirty paper interference.
- Integer-Forcing Beamforming: Same procedure as in uplink, but first pre-invert messages $\tilde{\mathbf{M}} = \mathbf{A}_p^{-1}\mathbf{M}$, then generate $\mathbf{X}$ from $\tilde{\mathbf{M}}$. Now, decoding $\mathbf{a}_k^{\mathsf{T}}\mathbf{X}$ will yield $\mathbf{m}_k$. (**Hong-Caire '13**).

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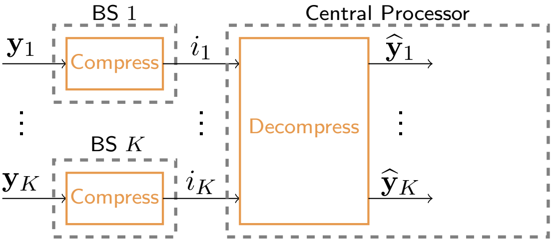
Uplink Compression $\equiv$ Distributed Source Coding



- Source Model: $\mathbf{Y} \sim \mathcal{N}(\mathbf{0}, \mathbf{I} + P\mathbf{H}\mathbf{H}^{\mathsf{T}})$.
- Joint Typicality Decoding (Berger-Tung): Search for jointly typical reconstructions $\hat{\mathbf{Y}}$.
- Successive Cancellation (Wyner-Ziv): Use $\hat{\mathbf{y}}_1, \ldots, \hat{\mathbf{y}}_{k-1}$ as side information for reconstructing $\mathbf{y}_k$.
- Integer-Forcing: First, reconstruct integer linear combinations of the sources $\mathbf{A}\hat{\mathbf{Y}}$, then solve for original sources. (**Ordentlich-Erez '17**)

Nested Lattice Quantization

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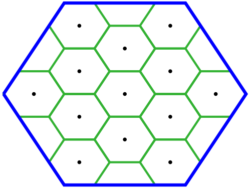

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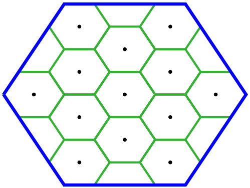

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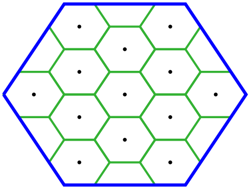

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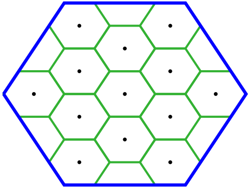

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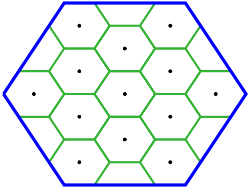

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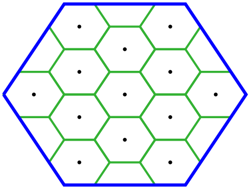

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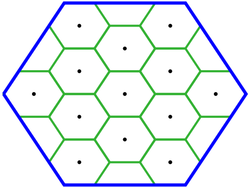

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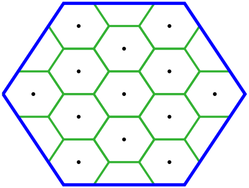

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- The achievable rate for IFSC with symmetric distortion is

$$R_{\mathsf{IFSC}} = \max_\ell \frac{1}{2}\log\left(\frac{\mathbf{a}_\ell^\mathsf{T}\left(\mathbf{K} + d\mathbf{I}\right)\mathbf{a}_\ell}{d}\right)$$

Select Integer Combinations to Minimize Effective Variance

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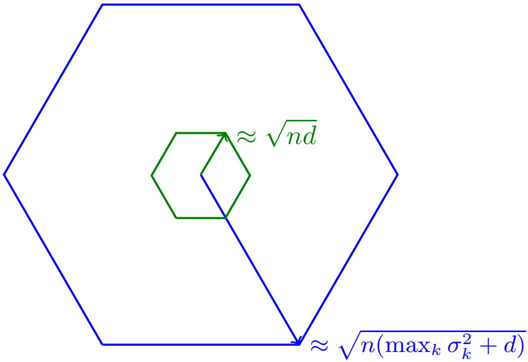

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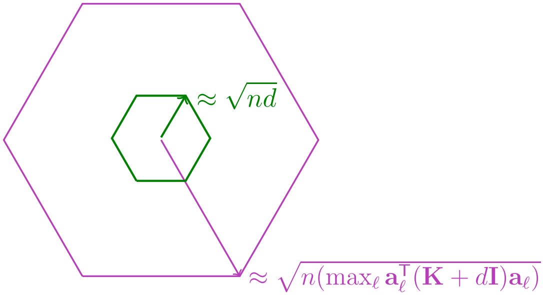

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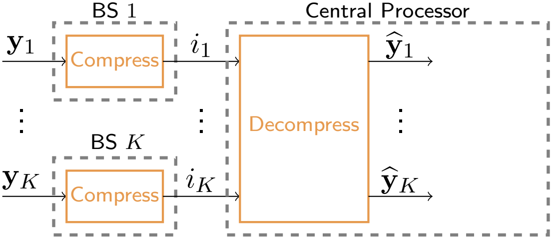

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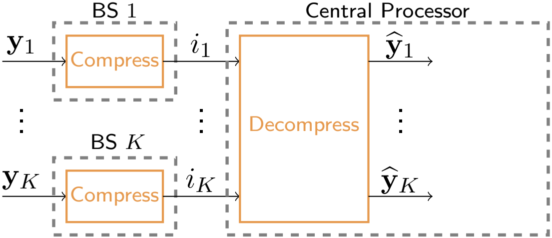

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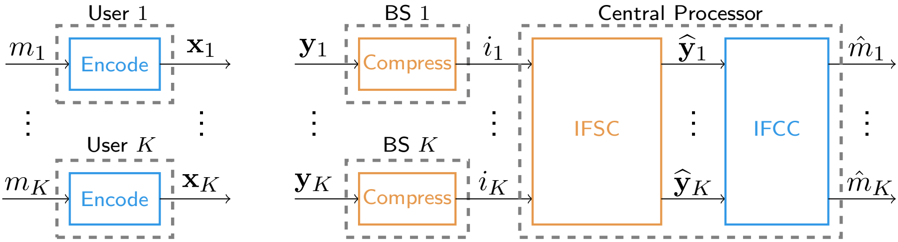
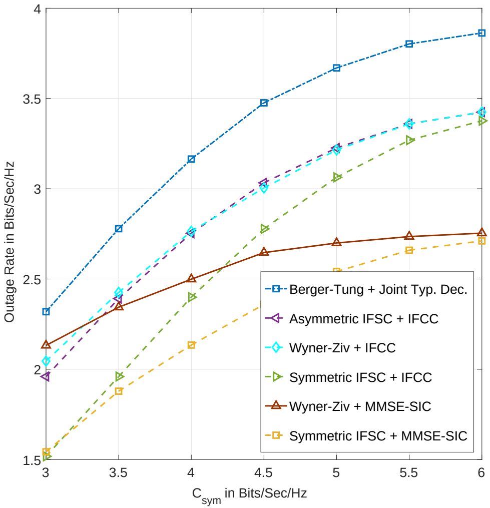
Integer-Forcing Uplink



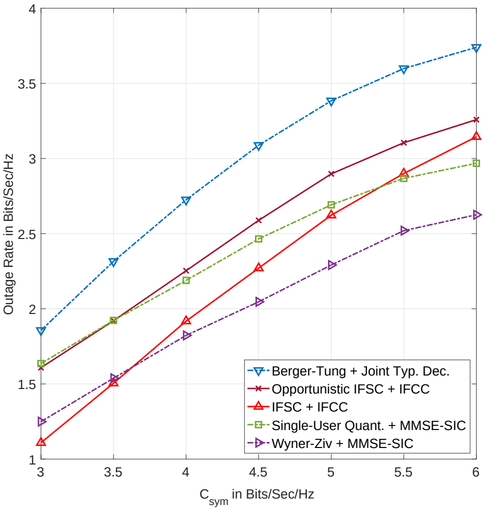
- **El Bakoury - Nazer '20:** Integer-forcing for uplink C-RAN.
- Users emit lattice codewords, no CSIT.
- BSs employ lattice quantization, rates set using global or local CSIR.
- Central processor reconstructs BS observations via IFSC, then recovers messages via IFCC.
- Constant-gap outage optimality (for global CSIR): $p_{\text{IF-CRAN}}(R - \Delta C) \leq p_{\text{optimal}}(R) + \gamma(\max\{K, L\})2^{-\Delta C/3}$

Uplink C-RAN Plot



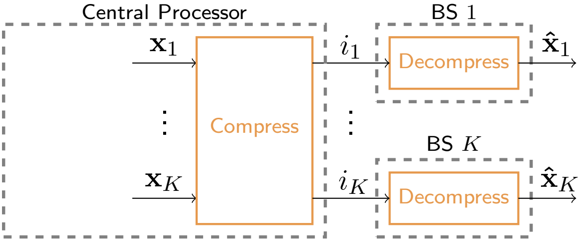
$K = 6$ users, $L = 6$ BSs, 5% outage, SNR $= 25$dB, global CSIR

Uplink C-RAN Plot



$K = 6$ users, $L = 6$ BSs, 10% outage, SNR $= 25$dB, local CSIR

Downlink Compression $\equiv$ Multivariate Compression



- Source Model: $\mathbf{X} \sim \mathcal{N}(\mathbf{0}, P\mathbf{I})$
- Joint Typicality Encoding: Search for jointly typical $\hat{\mathbf{x}}_1, \ldots, \hat{\mathbf{x}}_K$ that achieves quantization noise target covariance matrix $\mathbf{\Omega}$ and distortion constraints. (**Park et al. '13**)
- Integer-Forcing: Same procedure as integer-forcing source coding, but apply the inverse integer matrix prior to compression, $\mathbf{A}^{-1}\mathbf{X}$. Now, recovering $\mathbf{A}\hat{\mathbf{X}}$ will yield an estimate of $\mathbf{X}$ with quantization noise covariance matrix $d\mathbf{A}\mathbf{A}^{\mathsf{T}}$.

Integer-Forcing Multivariate Compression (with Symmetric Distortion)

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$$\hat{\mathbf{s}}_\ell = \left[\boldsymbol{\lambda}_\ell - \sum_k a_{\ell,k}\mathbf{u}_k\right] \bmod \Lambda_C \overset{\text{w.h.p.}}{=} \mathbf{s}_k + \sum_k a_{\ell,k}\mathbf{q}_k$$

where $\mathbf{q}_k = -[\mathbf{v}_k + \mathbf{u}_k] \bmod \Lambda_F \sim \text{Unif}(\mathcal{V}_F)$ and the quantization noise matrix has effective covariance matrix $d\mathbf{A}\mathbf{A}^{\mathsf{T}}$.

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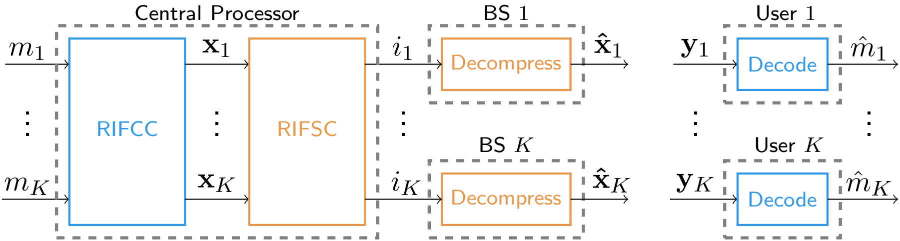
- As before, Λ_F determines the distortion level d (i.e., $\sigma^2(\Lambda_F) = d$).
- Λ_C must contain $\mathbf{s}_k + \sum_k a_{\ell,k}\mathbf{q}_k \ \ \forall k$ so $\sigma^2(\Lambda_C) = \max_\ell \sigma_\ell^2 + d\mathbf{a}_\ell^{\mathsf{T}}\mathbf{a}_\ell$
- The achievable rate for IFMC with symmetric distortion is

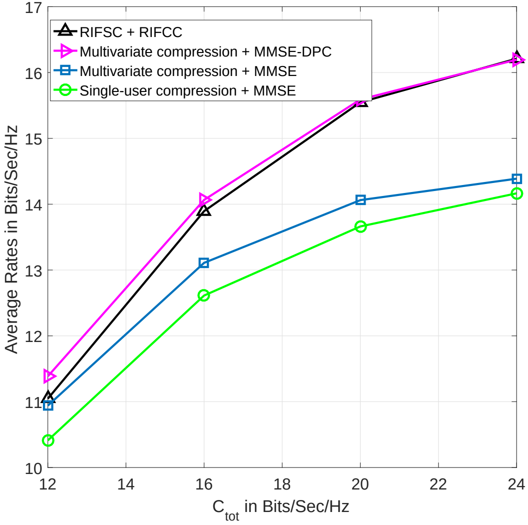
$$R_{\text{IFMC}} = \max_\ell \frac{1}{2}\log\left(\frac{\sigma_\ell^2 + d\mathbf{a}_\ell^{\mathsf{T}}\mathbf{a}_\ell}{d}\right)$$

Integer-Forcing Downlink



- **El Bakoury - Nazer '20:** Integer-forcing for downlink C-RAN.
- BSs send sequence corresponding to quantization index.
- Users employ single-user decoding.
- Central processor encodes messages via RIFCC, then quantizes codewords via RIFSC to correlate the quantization noise.
- RIFSC: Same sum rate as joint typicality multivariate compression.
- Uplink-downlink duality with uplink integer-forcing C-RAN (with full CSI and total power constraint).

Downlink C-RAN Plot



$K = 4$ users, $L = 4$ BSs, SNR $= 30$dB, full CSI

Overview: Integer-Forcing Channel and Source Coding

	Channel	Source
Uplink		
Downlink		

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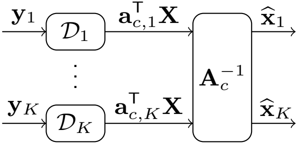
Uplink	 $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ IF Equalization [Zhan et al. '14]	
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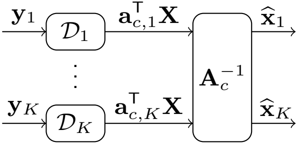
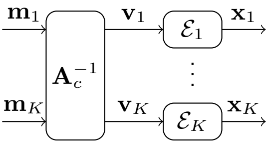
Uplink	 $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ IF Equalization [Zhan et al. '14]	
Downlink	 $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ IF Beamforming [Nam-Caire '13]	

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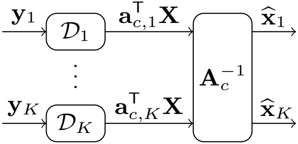
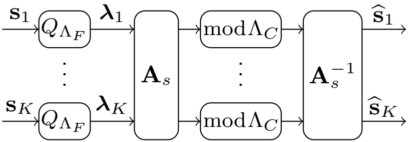
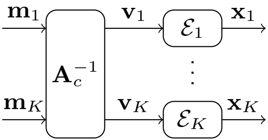
Uplink	 $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ IF Equalization [Zhan et al. '14]	 $\mathbf{S} \sim \mathcal{N}(\mathbf{0}, \mathbf{K}_{SS})$ IF Distributed SC [Ordentlich-Erez '15]
Downlink	 $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ IF Beamforming [Nam-Caire '13]	

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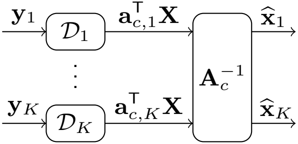
Channel | Source

Uplink

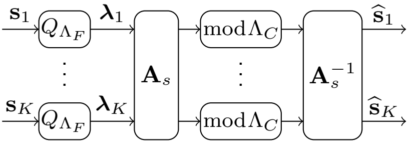
Channel



$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$$

IF Equalization [Zhan et al. '14]

Source

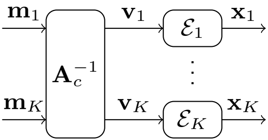


$$\mathbf{S} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{K}_{SS}\right)$$

IF Distributed SC [Ordentlich-Erez '15]

Downlink

Channel



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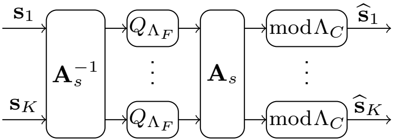
IF Beamforming [Nam-Caire '13]

Source



$$\widehat{\mathbf{S}} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{K}_{SS} + \mathbf{A}_s\mathbf{D}\mathbf{A}_s^{\mathsf{T}}\right)$$

IF Multivariate Compression [New]

Uplink:

- Joint typicality coding operates within a constant gap of the sum capacity (with full CSI) and symmetric outage capacity (**Zhou-Yu '14 , Ganguly-Hong-Kim '19**). Capacity known if relays restricted to oblivious operation (**Aguerri et al. '17**).
- Sequential coding attains a constant gap to the sum capacity under a sum fronthaul rate constraint. (**Zhou et al. '16**)
- Integer-forcing attains a constant gap to the symmetric outage capacity.

Downlink:

- Joint typicality coding operates within a constant gap of the sum capacity (**Liu-Patel-Yu '16, Ganguly-Hong-Kim '19**). Capacity for special cases (**Bidokhti-Kramer '16**).
- Sequential coding can reach a constant gap to the sum capacity under a sum fronthaul rate constraint (**Patil-Yu '18**).
- Integer-forcing performance not known, but we conjecture that a constant gap to the sum capacity is also attainable.