

**Problem 1***10 points*

For each of the following parts, indicate whether the statement is always true or it can be false by clearly writing “True” or “False.” Briefly explain the reasoning behind your answer for partial credit (in case your choice is wrong). Diagrams are welcome. Throughout the problem, you may assume that  $A$ ,  $B$ , and  $C$  are events with  $\mathbb{P}[A] > 0$ ,  $\mathbb{P}[B] > 0$ , and  $\mathbb{P}[C] > 0$ .

- (a)  $\mathbb{P}[A \cap B] + \mathbb{P}[B \cap C] = \mathbb{P}[B]$
- (b) If  $\mathbb{P}[A] + \mathbb{P}[B] = \mathbb{P}[A \cup B]$ , then  $\mathbb{P}[A \cap B] = 0$ .
- (c) If  $A$  and  $C$  are independent, then  $A$  and  $B$  are conditionally independent given  $C$ .
- (d) If  $A \cap B \cap C = \phi$ , then  $\mathbb{P}[A \cup B \cup C] = \mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C]$ .
- (e) If  $\mathbb{P}[A] = 0.4$ ,  $\mathbb{P}[B^c] = 0.4$ , and  $\mathbb{P}[(A \cup B)^c] = 0.24$ , then  $A$  and  $B$  are independent.

**Problem 2***10 points*

For each of the following parts, indicate whether the statement is always true or it can be false by clearly writing “True” or “False.” Briefly explain the reasoning behind your answer for partial credit (in case your choice is wrong). Diagrams are welcome. Throughout the problem, you may assume that  $X$  is a discrete random variable with PMF  $P_X(x)$  and CDF  $F_X(x)$ .

- (a) If  $F_X(a) = 1$ , then  $\mathbb{E}[X] \leq a$ .
- (b)  $\mathbb{P}[X^2 > 0] = 1 - P_X(0)$ .
- (c) If  $\mathbb{E}[X] = a$ , then  $\mathbb{E}[X^2] \leq a^2$ .
- (d) If  $\mathbb{E}[X] = 2$  and  $\mathbb{E}[X^2] = 4$ , then  $\text{Var}[X] = 0$ .
- (e)  $\mathbb{P}[a < X < b] = \mathbb{P}[X > a] \mathbb{P}[X < b]$  for  $a < b$ .

**Problem 3** Please complete the following quick calculations.

*16 points*

(a) Let  $X$  be Poisson(2).

Calculate  $\mathbb{E}[2X + 1]$  and  $\mathbb{E}[(2X + 1)^2]$ .

(b) Let  $X$  be Bernoulli(1/3).

Calculate  $\mathbb{E}[X + 1]$  and  $\mathbb{E}\left[\frac{1}{X + 1}\right]$ .

(c) Let  $A$ ,  $B$ , and  $C$  be independent events with  $\mathbb{P}[A] = \mathbb{P}[B] = \mathbb{P}[C] = \frac{2}{3}$ .

Calculate  $\mathbb{P}[A \cup B]$  and  $\mathbb{P}[A \cup B \cup C]$ .

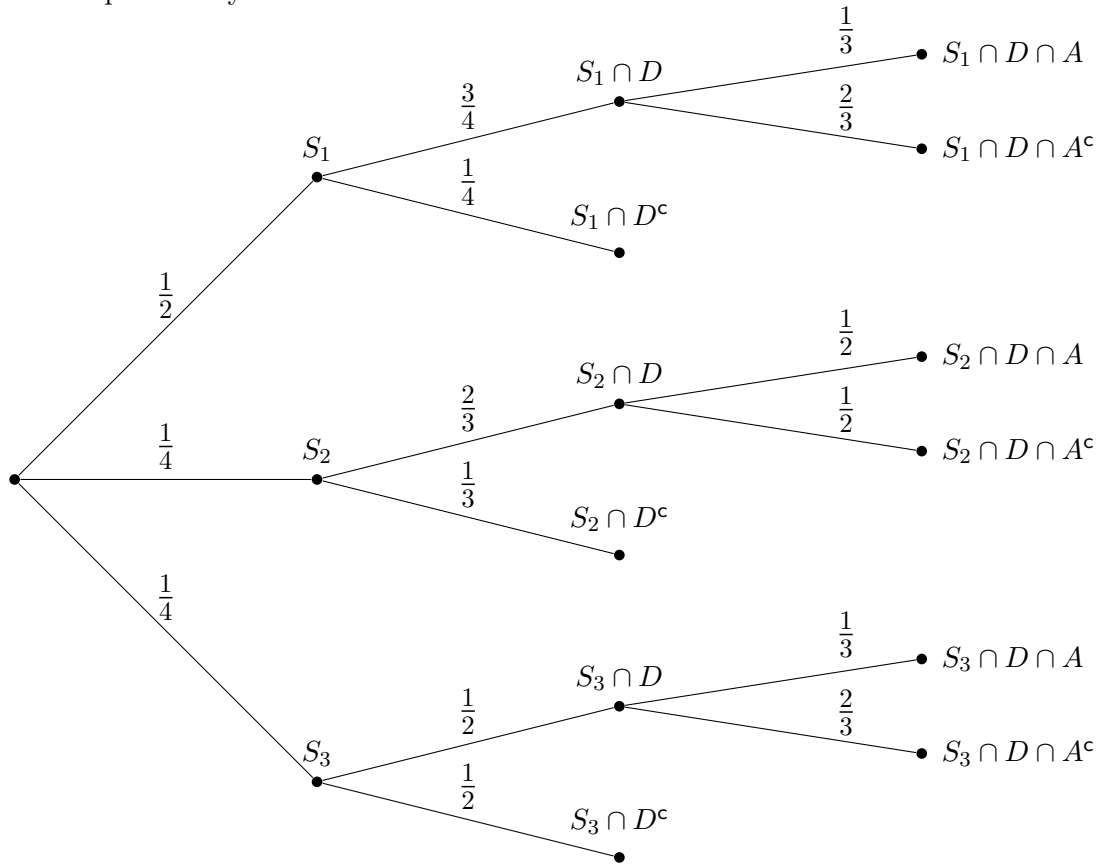
(d) Let  $A$  and  $B$  be events with  $\mathbb{P}[A \cap B] = \frac{5}{8}$  and  $\mathbb{P}[A^c \cap B] = \frac{1}{8}$ .

Calculate  $\mathbb{P}[B]$  and  $\mathbb{P}[A|B]$ .

**Problem 4**

16 points

You are an archaeologist embarking on an expedition to find a Argentinosauros dinosaur fossil to complete the local museum's collection. You randomly decide to visit one of three dig sites, where  $S_i$  for  $i \in \{1, 2, 3\}$  is the event that you visit the  $i^{\text{th}}$  site. Let  $D$  be the event that you discover a dinosaur fossil at your site, and let  $A$  be the event that, upon careful examination, it shown to be a Argentinosauros fossil. The probabilities of these events are modeled via the conditional probability tree below.



- What is the probability that you do **not** find a dinosaur fossil?
- What is the probability that you visit Site 1 and find a Argentinosauros fossil?
- Given that you find a Argentinosauros fossil, what is the probability that you were **not** at Site 1?
- Given that you do **not** visit Site 1, what is the probability that you find a dinosaur fossil?

**Problem 5***16 points*

On your first day at the job at the Pear Corporation, you have been asked to work out a few quantities for the random variable  $X$  with PMF

$$P_X(x) = \begin{cases} \frac{3}{5} & x = 0 \\ \frac{1}{5} & x = 1 \\ 0 & x = 2 \\ \frac{1}{5} & x = 3 \end{cases}$$

- (a) Make a sketch of the PMF. (Don't forget to label the axes.)
- (b) Calculate  $\text{Var}[X]$ .
- (c) Calculate  $\mathbb{E}[2^X]$ .
- (d) For  $B = \{1, 3\}$ , calculate  $\mathbb{E}[X|B]$ .

**Problem 6***16 points*

You are running a jellybean factory and producing tiny pouches that contain four jellybeans. Each jellybean is equally likely to be one of four colors, red, blue, green, yellow, independently of the other jelly beans.

- (a) What is the probability that all four jellybeans in a pouch are green?
- (b) What is the probability that all four jellybeans in a pouch are the same color?
- (c) What is the probability of getting all four colors in a single pouch?
- (d) Given that there are no blue jellybeans in a pouch, what is the probability that there are no green jellybeans?

**Problem 7***16 points*

In your small hometown, the number of Wikipedia searches for a “CD Player” in a given day can be modeled as a discrete random variable  $X$  which is Poisson(2).

- (a) What is the probability that  $X$  is less than 2?
- (b) Given that  $X$  is less than 4, what is the probability that  $X$  is less than 2?
- (c) Given that  $X$  is less than 4, what is the conditional expected value of  $X$ ?
- (d) Assume that searches across days are independent from one another. Let  $Y$  be the number of days until the first day with 2 or more searches. What kind of random variable is  $Y$ ? (Don't forget the parameters.)