- **Example:** Measure the radius of 100 cells and obtain a sample mean radius of $M_{100} = 5.10\mu m$. From prior studies, we know the standard deviation is $\sigma = 0.53\mu m$.

→ Find a confidence interval for the mean with confidence level 0.95.

Since variance is known, $[M_n - \epsilon, M_n + \epsilon]$ with $\epsilon = \frac{\sigma}{\sqrt{n}} Q^{-1}(\frac{\alpha}{2})$ is a confidence interval for the mean with confidence level $1-\alpha$.

Solve for $\frac{\alpha}{2}$: $1-\alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

Lookup $Q^{-1}(\frac{\alpha}{2})$: MATLAB $Q^{-1}(\frac{\alpha}{2})$ = qfuncinv$(\frac{\alpha}{2})$

$Q^{-1}(0.025)$ = qfuncinv(0.025) = 1.96

Solve for ϵ: $\epsilon = \frac{0.53\,\mu m}{\sqrt{100}} \cdot 1.96 = 0.10\mu m$

Either format is OK.

$[5.10\mu m \pm 0.10\mu m] = [5.00\mu m, 5.20\mu m]$ is a confidence interval for the mean with confidence level 0.95.

• **Example:** Measure the radius of 100 cells and obtain a sample mean radius of $M_{100} = 5.10\mu m$ and a sample variance of $V_{100} = 0.80\mu m^2$.

→ Find a confidence interval for the mean with confidence level 0.95.

Since the variance is unknown, $[M_n - \epsilon, M_n + \epsilon]$ with $\epsilon = -\frac{\sqrt{V_n}}{\sqrt{n}} F_{T_{n-1}}^{-1}\left(\frac{\alpha}{2}\right)$ is a confidence interval for the mean with confidence level $1-\alpha$.

Solve for $\frac{\alpha}{2}$: $1-\alpha = 0.95 \Rightarrow \frac{\alpha}{2} = 0.025$

Lookup $F_{T_{n-1}}^{-1}\left(\frac{\alpha}{2}\right)$: MATLAB $F_{T_{n-1}}^{-1}\left(\frac{\alpha}{2}\right) = \text{tinv}\left(\frac{\alpha}{2}, n-1\right)$

$n = 100 \Rightarrow n-1 = 99$ $\quad F_{T_{99}}^{-1}(0.025) = \text{tinv}(0.025, 99) = -1.98$

Solve for ϵ: $\epsilon = -\frac{\sqrt{0.80\mu m^2}}{\sqrt{100}}(-1.98) = 0.18\mu m$

$[5.10\mu m \pm 0.18\mu m] = [4.92\mu m, 5.28\mu m]$ is a confidence interval for the mean with confidence level 0.95.

• <u>Example:</u> Measure the radius of 100 cells and obtain a sample mean radius of $M_{100} = 5.10\,\mu m$ and a sample variance of $V_{100} = 0.80\,\mu m^2$.

→ Find a confidence interval for the variance with confidence level 0.95.

$[\beta_1 V_n, \beta_2 V_n]$ with $\beta_1 = \dfrac{n-1}{F^{-1}_{\chi^2_{n-1}}(1-\frac{\alpha}{2})}$ and $\beta_2 = \dfrac{n-1}{F^{-1}_{\chi^2_{n-1}}(\frac{\alpha}{2})}$ is a confidence interval for the variance with confidence level $1-\alpha$.

Solve for $\frac{\alpha}{2}$: $1-\alpha = 0.95 \Rightarrow \frac{\alpha}{2} = 0.025 \Rightarrow 1-\frac{\alpha}{2} = 0.975$

Lookup $F^{-1}_{\chi^2_{n-1}}(z)$: MATLAB $F^{-1}_{\chi^2_{n-1}}(z) =$ chi2inv(z, n-1)

$F^{-1}_{\chi^2_{99}}(0.975) =$ chi2inv(0.975, 99) $= 128.42$

$F^{-1}_{\chi^2_{99}}(0.025) =$ chi2inv(0.025, 99) $= 73.36$

Solve for β_1, β_2: $\beta_1 = \dfrac{99}{128.42} = 0.77$ $\quad \beta_2 = \dfrac{99}{73.36} = 1.35$

$[0.77 \cdot 0.80\,\mu m^2,\ 1.35 \cdot 0.80\,\mu m^2] = [0.62\,\mu m^2,\ 1.08\,\mu m^2]$ is a confidence interval for the variance with confidence level 0.95.